A critique of science reveals that science is an indispensable instrument by means of which man can improve the material conditions of life in a world which is open to progress.—CARROLL D. W. HILDEBRAND.

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A CRITIQUE OF SCIENCE*

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"Accuracy is the soul of science."

INTRODUCTION

It is well to remember that all thinking rests upon assumption and presupposition. Thinking about science is no exception. If, in this study, attention can be centered upon the nature of science rather than upon the conclusions to which science comes respecting particular subject matter, our purpose can best be realized. Science may properly be defined as an attempt to discover and to formulate the characteristic laws of some selected domain of experience. It is organized knowledge within space-time. Formal sciences (logic and mathematics) should be distinguished from empirical sciences which, in turn, may be classified as physical sciences (physics, chemistry, *et cetera*) and social sciences (history, sociology, economics, *et cetera*).

Since a critique of science is essentially a philosophical enterprise, it is well to define the science of philosophy. A. N. Whitehead defined philosophy as an "endeavor to frame a coherent, logical, necessary system of general ideas in terms of which every element of our experience can be interpreted." This distinction between a philosophy of science and the science of philosophy must be made since this study is concerned with the meaning and evaluation of science. A critique of science is, then, a philosophical interpretation of science.

* An address delivered at the opening meeting of the Indiana Academy of Science on November 10, 1944. Submitted for publication in SCHOOL SCIENCE AND MATHEMATICS upon request of the General Biology editor. J.E.P.

There are at least two important reasons for the significance of this study. They are (1) science is the world-view of the modern educated man, and (2) this is a matter-of-fact civilization which, in Thorstein Veblen's judgment, is in danger of producing a crisis that can result in a social breakdown because its emphasis is upon the material expression of civilization to the neglect of other values. The purpose of a critique of science is to determine the place of science in modern civilization and culture.

The method by which this study seeks to establish a critique of science is that of formulating the fundamental assumptions and presuppositions which underlie scientific thinking as such. We are then in a position to state relevant conclusions concerning the nature of science. This is what we mean by a critique of science.

Theses

My first thesis asserts that it is not the refutation of science but its proper delimitation that is required in order to understand the meaning of science. The history of science, philosophy, and religion[1] is replete with evidence to demonstrate that the so-called conflict between science and religion was due to the failure to understand this distinction. Two illustrations will suffice. The Copernican Revolution in astronomy and the Darwinian theory of evolution in biology were scientific achievements too often met by scientific and religious dogmatists who sought to refute science whenever it appeared to contradict the teachings of religion and morality instead of coming to a harmonious understanding of the limits of science.

Fortunately, we have overcome the history of the warfare between science and religion as it has been erroneously called. It has become clear that, in the nature of the case, no such conflict could exist. Actually, there was misunderstanding due to conflict between certain scientific theories about a world-view and a life-view and certain theological theories concerning a world-view and a life-view. Once the claims of these conflicting theories were adjusted, it was seen that truth is a seamless robe and that science and religion do not contradict each other. What we have achieved from the history of science for our day is the insight that the science of a religious man can remain

[1] Cf. Dampier, Sir William, *A History of Science and Its Relations with Philosophy and Religion*, 3rd ed., revised and enlarged. Macmillan, New York. White, Andrew D., *A History of the Warfare of Science with Theology in Christendom*, 2 vols. N. Y. Appleton, 1903, and others.

scientific and the religion of a scientific man can remain religious. This thesis contends that science cannot be legitimately refuted but that it ought to be delimited. Consequently, the remaining theses of this critique are all delimitations of science.

The first delimitation of science is revealed in the fact that science is abstract as well as concrete. By initial predication, science "bags its universe piece-meal," that is, its knowledge consists in the discovery of law or order within some selected domain of experience. Each science, such as chemistry, physics, and the like, discovers, describes, and classifies the facts which constitute its subject matter. It then proceeds to formulate whatever laws, hypotheses, and theories are necessary to explain these facts. To be sure, all sciences have their border-line problems, such as the problems of geo-physics, bio-chemistry, psycho-physics, socio-economics, and others, which reveal the interpenetration of the sciences but in no wise destroy the uniqueness of each science.

Scientific knowledge, then, is knowledge seen out of relationship to experience as a whole, which in no wise weakens science. It is the sole condition under which science can fulfill its purpose, namely to provide the power of control over environment. Science achieves this control by explaining how things go together in the order of nature. Why there is an order of nature and what purpose it ultimately expresses constitute no essential part of the scientific enterprise. This is what is meant by saying that scientific knowledge is in nature abstract despite its matter-of-fact content. Here we encounter not a contradiction but a scientific paradox which asserts that science is at once abstract and concrete.

Our second delimitation of science emerges with the recognition of the presuppositions upon which science rests. Science presupposes the existence of mind, both individual and social. Without this assumption, no scientific datum could be known to exist, much less could it be interpreted. Science presupposes universals, for the denial of universals means, among other things, that no scientific law could either be generalized or be assumed to have reality. Science also presupposes ideals. Even though science is indifferent to all ideals save one, she presupposes and insists upon loyalty to the ideal of science including freedom to inquire, objectivity, and the love of scientific truth. Science presupposes values. Although science may be neutral to all other values, she recognizes the value of science, in both its

theoretical and its practical aspects. Moreover, science presupposes a method, empirical and analytic. In fact, it is method rather than any particular subject matter that constitutes science. Finally, science presupposes the existence of the external world. The nature of the physical world involves us in the metaphysics of physics. Other theses will further delimit science at this point. We are concerned here only with the existence of the external world as a presupposition of science.

Thirdly, these presuppositions call for the delimitation of science which asserts the distinction between scientific and extra-scientific types of experience. That a scientific approach may be made to any fact in the universe does not preclude the necessity for recognizing both the possibility and the desirability of an extra-scientific approach to those same facts. If we are correct in stating the presuppositions of science, then science itself presupposes an extra-scientific order of experience. Art, religion, morality, and philosophy comprise aspects of extra-scientific experience. The existence of these non-scientific domains of experience is at least as certain as is the existence of scientific experience. This distinction is logical as well as psychological. Failure to recognize extra-scientific types of experience obscures the relation between science and philosophy and religion; or it may result in the unjustified attempt to identify philosophy and religion with science in which case we attempt a philosophy of science and a religion of science which raises serious questions for coherent thought.

Fourthly, a delimitation of science must be made explicit in connection with its presupposition of the existence of the external world of physical nature. Metaphysical implications are unavoidably present at this point. But that science may be substituted for philosophy and religion would be an unjustified extension of the domain of scientific knowledge. Whenever science is offered as a substitute for either philosophy or religion the result is to regard science as metaphysics or to regard it as positivistic and agnostic and, therefore, as being anti-metaphysical in nature. Both of these results are to be rejected and to be explained as the direct result of our failure properly to delimit or understand the scope of scientific inquiry.

Where science is substituted for metaphysics in philosophy and religion, the result is some form of materialism known as energism, metaphysical behaviorism, or physical realism. The philosophical naturalism of the nineteenth century like that of

the twentieth, so far as it is materialistic, rests upon this logical fallacy that the sole and only metaphysics which scientific data will support is metaphysical naturalism. On the other hand, the substitution of science for philosophy and religion may take that form of philosophical naturalism which is known as scientific positivism or it may take the form of agnosticism. If science be made a synonym for positivism of the sensationalistic type, all knowledge of the nature of ultimate reality is dogmatically denied. If it be identified with critical positivism, the result is agnosticism which may leave the question of the nature of ultimate reality a matter of suspended judgment and wholly undetermined. Neither science as metaphysics nor science as philosophical skepticism and agnosticism is a coherent and synoptic view of human experience. In any case, philosophy is needed either to refute sense metaphysics or to transcend skepticism, positivism, and agnosticism. In other words, the empire of knowledge must be divided between science, philosophy, art, morality, and religion. Science is not all.

Fifthly, science, together with its method (experiment, analysis, generalization, description, system, and order), is a creation of the human mind and, as such, constitutes a scientific ideal. It has been shown that science presupposes ideals in the sense that it loyally supports and realizes the scientific ideal. Science, however, is neutral if not indifferent to all other ideals. This limitation of science reveals that science as such must derive the ideal which controls the use made of it by the individual and society from some source other than and beyond itself. The tragic era of civilization in which our lives are cast amply illustrates the force of this contention. Carl Dreher,[2] an electrical engineer, has shown that science and technocracy can solve the socio-economic problem. But a choice must be made between some form of totalitarian collectivism (communist, fascist, nazi, or falangist) which is based upon war and economic imperialism, and democratic collectivism which ought to be based upon a world welfare economy and economic co-operation. Science can serve and is now serving these conflicting ideals with equal efficiency. In the same way, Charles E. Merriam,[3] a political scientiest, shows that the assumptions which underlie political democracy constitute an ideal which is necessary for the direction of an expanding national economy so that science and tech-

[2] Cf. *The Coming Showdown*, N. Y. Little, Brown, 1942.

[3] Cf. *The New Democracy and the New Despotism*, N. Y. McGraw-Hill, 1939.

nocracy may be secured in the service of a constructive ideal of society rather than in the service of its ruthless exploitation.

Here we come upon a most significant limitation of science, namely, that, while superior to such other sources of authority for human ideals as custom, tradition, desire, wealth, and law, science can never be regarded as an adequate source of authority for human ideals. It can only tell man what he can do, never what he ought to do. Since science is neutral to ideals, the ideal which directs its employment is derived from one's philosophy of life (philosophy and/or religion) and is not generated from within science itself. Thus, science can make man "better off" but it cannot make him better.

Sixthly, another limitation of science is revealed in the fact that science is *wertfrei*. This does not mean that science is valueless, but that it is independent of value. Science does not regard its results in the light of their bearing upon aesthetics, morality, or religion. In fact, science is descriptive and whatever value attaches to it derives from knowing the facts concerning what is. For example, psychology and sociology describe the values which have been and now are approved. They do not tell us what value or system of values ought to have been approved or ought to be approved now. Neither do they tell us why these values should be approved. Science deals with matters of fact which make up our existential judgments concerning the space-time world. Value is a fact as much as is electricity or light, but it is an order of fact calling for a distinction between physical fact and value. The value of science is in no wise impaired when we recognize that it is value-free or independent of value because science has been and is of enormous value to both civilization and culture. In a recent letter inviting membership in the American Association for the Advancement of Science, F. R. Moulton, its permanent secretary, said,

> "It is obvious to every one that the outcome of the war will depend on science—on physics and chemistry and other physical sciences; and likewise on psychology and medicine and all the biological sciences. It is also obvious to thinking persons that the long future of humanity will depend equally upon science—both the natural sciences and the social sciences."

However great the practical and utilitarian value of science may be it has also a cultural value. The practical value of science in its control over nature and in terms of commercial value must not overshadow what scientific data have meant by way of enriching man's world-view and his life-view through purging

religion of superstition and philosophy of mere speculation. This admitted, we must not obliterate the distinction between the possession of scientific inventions and the ends which these inventions are made to serve. This distinction calls for a system of values with which science as such has no concern.

Lastly, science is delimited by the nature of scientific method which is empirical, laboratory in the physical sense, and analytical. By empirical is meant that scientific judgments are existential in nature, that is, they organize knowledge within space-time. By the laboratory method in the strict sense is meant the study of physical reactions. Induction through scientific analysis means the attempt to analyze and to synthesize the units of structure which constitute the essential nature of any given phenomenon under scientific investigation on the assumption that the whole is equal to the sum of its parts. Whether compounds, organisms, or wholes may not possess traits and characteristics as wholes which are more than the sum of their parts is an open question in the present state of science. It stimulates the quest for some method which will correct the defect of analysis despite the fact that analysis is generally regarded to be the most improved method of science at the present time. A scientist and statesman of the distinction of Jan Christian Smuts[4] recognizes this limitation of analysis. In order to correct it, he recommends "holism" or what we may call the synoptic or philosophical use of the reason. In this way, the imponderable facts of human experience such as the theory of induction, universals, ideals, values, and consciousness or mind are included along with physical processes in our interpretation of nature.

Conclusions

1. We need a critique of science not to refute but to delimit science.

2. Science can provide a valuable knowledge of the order of physical nature but, because of the abstract nature of science, it can explain neither why such order prevails nor the purpose which it ultimately fulfills.

3. A critique of science which reveals the limitations of science also reveals that man's total well-being depends upon the recognition of extra-scientific types of experience such as art, philosophy, religion, and morals as well as of science.

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[4] Cf. *Holism and Evolution*, 3rd ed. N. Y. Macmillan, 1936.

4. A critique of science reveals that science is an indispensable instrument by means of which man can improve the material conditions of life in a world which is open to progress.

5. The neutrality of science to both ideals and values other than the scientific ideal and the value of science may lead, and sometimes actually has led, to the hypothesis of the indifference of nature to human hopes and aspirations.

6. Science is limited in scope and method in such way that the ideal which directs its employment for benevolent and socially approved ends must be derived from some source outside science.

7. A critique of science reveals the perils of science no less than its values. We mention three of them.

(a) The growth of science with its discoveries and inventions has produced the industrial machine age in which man's world has been progressively depersonalized and human life cheapened.

(b) There is good reason to believe that the present breakdown in civilization and culture is due to man's acute scientific orientation within a this-worldly environment and a consequent weaning away from the spiritual laws of the universe.

(c) The abuse of science as the embezzlement of power is a direct threat to democracy.

8. There is a cultural value of natural knowledge which may contribute to the enrichment of a philosophical and religious world-view and life-view.

BATTELLE MEMORIAL INSTITUTE'S RESEARCH PROGRAM

To help America maintain its top-flight position in science, an expansion of Battelle Memorial Institute's program of research education is being planned, Clyde Williams, Battelle director, announced today. Mr. Williams pointed out that the program will be of special interest to returning veterans who can qualify for training as research workers in the sciences.

The program, which will be directed at the graduate level of education, is an expansion of the plan which has been markedly successful thus far with Ohio State University.

Thousands of veterans have become aware of the importance of technology from their battlefield experiences and will be interested in scientific or engineering careers. They are learning that scientific manpower is one of the country's key assets, and that the supply of such men is now, and will remain for years, critically short. Battelle Memorial Institute, in cooperation with other educational institutions, Williams declared, will give qualified veterans the finest possible training in scientific research on the professional level.

PREDICTING ACCOMPLISHMENT IN PLANE GEOMETRY

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In one of the more recent reviews of investigations dealing with prediction of achievement in secondary school mathematics Douglass[1] points out that the most frequent type of investigation has concerned itself with the relationship between scholastic success and other factors such as intelligence test scores and school marks in previous subjects. On the basis of his review he concluded: "(1) Achievement in algebra and geometry may be predicted with a fair degree of accuracy only. (2) Achievement cannot be predicted satisfactorily from any one variable for the purpose of homogeneous grouping or definite advice relative to taking or not taking geometry. (3) Achievement is best predicted by a combination of the following variables—a good prognostic test, I.Q., and average mark in previous year or two years of school work." In ranking these variables for prediction of success, Douglass places the prognostic test first and the average mark on previous school year second. He states, however, that there is little difference in their value for prognostic purposes. Following these he places I.Q., previous teachers' estimate of mathematic ability, mental age, achievement test or mark in previous year's work in mathematics, chronological age, and character rating.

The present study was conducted at Harris High School, Petersburg, Illinois, during the school year 1940–41. Of the 315 pupils attending this four-year high school, 38 enrolled in plane geometry. The school draws its enrollment from the junior-high school and from one and two room rural schools within the county. The high-school curriculum includes four years of mathematics including practical mathematics, but only two semesters of mathematics are required for graduation. Pupils may enroll in geometry after one year of algebra or practical mathematics or both. All of the pupils used in this study, however, had taken algebra and three had had both practical mathematics and algebra. There is no guidance program in operation in this school; consequently, the pupils were given only scant aid in selecting their subjects.

[1] Harl R. Douglass, The Prediction of Success in High School Mathematics, *The Mathematics Teacher*, 1935, *28*, pp. 489–504.

In the geometry class under consideration, the chronological ages ranged from 14 to 17 and the intelligence quotients from 84 to 131 with a mean of 106. One pupil was in the 12th grade, two in the eleventh and the remainder were in the tenth grade.

The purpose of the study was to determine the relative effectiveness of the following factors in predicting achievement in geometry: (1) scores on the Stewart-Davis[2] Test of Ability in Geometry; (2) intelligence quotients on the Otis Self-Administering Test of Mental Ability; (3) eighth-grade arithmetic marks; and (4) final marks in algebra. The criteria of achievement in plane geometry were: (1) an objective teacher-made achievement test in geometry given at the end of the first and second semesters, and (2) the Orleans Plane Geometry Test given at the end of the second semester.

Each of these teacher-made objective tests was correlated with the Orleans Plane Geometry Achievement Test. The scores on the objective teacher-made test administered at the end of the first semester correlated with the scores on the Orleans Plane Geometry Test 0.87. The coefficient of correlation between scores on the objective teacher-made test given at the end of the second semester and the Orleans Achievement test was 0.81.

Each criterion of prediction was correlated with the Orleans Plane Geometry Achievement Test and with a composite criterion consisting of the two teacher-made tests and the Orleans test. The raw scores on the achievement test were made comparable by the T-score method; an average of these was used in obtaining a composite score of the three tests.

A summary of the results is presented in Table I which shows the predictive factors both singly and compositely, as correlated with the Orleans Plane Geometry Achievement Test; and the combined scores of the two teacher-made objective tests and the Orleans Achievement Test.

The trends evident in Table I are:

1. The best single prediction criterion of the four studied is the Stewart-Davis Test of Ability in Geometry. This test, however, is not appreciably more accurate than the intelligence quotients obtained by the use of the Otis Self-Administering Test of Mental Ability.

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[2] This test consisting of four parts: (1) identification, (2) comparison by superposition, (3) logical selection, and (4) problem analysis, was administered at the beginning of the year before any work in plane geometry was attempted. Bureau of Educational Research, University of Colorado, 1940.

TABLE I. CORRELATION OF PREDICTION FACTORS WITH ACHIEVEMENT SCORES

Predictive factors	Correlation with Orleans Achievement Test	Correlation with Composite Achievement Scores
1. Stewart-Davis test	$r=0.88\pm0.02$	$r=0.89\pm0.02$
2. Algebra final marks	$r=0.78\pm0.04$	$r=0.87\pm0.02$
3. Intelligence quotients	$r=0.85\pm0.029$	$r=0.86\pm0.029$
4. Eighth grade arithmetic marks	$r=0.59\pm0.096$	$r=0.59\pm0.096$
5. Stewart-Davis test and algebra marks	$r=0.89\pm0.02$	$r=0.95\pm0.01$
6. Stewart-Davis test and intelligence quotients	$r=0.86\pm0.029$	$r=0.91\pm0.02$
7. Algebra final marks and intelligence quotients	$r=0.85\pm0.029$	$r=0.91\pm0.02$
8. Stewart-Davis test scores, algebra final marks, and intelligence quotients	$r=0.88\pm0.02$	$r=0.93\pm0.01$

2. Marks in arithmetic have limited value in predicting achievement in plane geometry.

3. The best prediction criterion is a combination of the Stewart-Davis test and algebra marks.

4. A combination of the Stewart-Davis Test and intelligence quotients has about the same predictive value as final examinations in algebra and intelligence quotients.

A statement by Rogers[3] is pertinent: "Predictions can never be absolute and mistakes will certainly be made. All we can do is to state whether a pupil's chances of succeeding are great or small." It is still possible to predict those students almost certain to fail by the use of algebra marks and a valid prognostic test. A low score does not necessarily mean failure, but it does mean that the individual has a very small chance to be successful in the subject.

[3] Agnes Low Rogers, "Psychological Test of Mathematical Ability and Educational Guidance," *The Mathematics Teacher*, 1923, *16*, 193–205.

"FIBERS"

This is the subject discussed by Professor Palmer and his assistants in the November issue of the *Cornell Rural School Leaflet*. A copy of this journal should go to every rural school. It is always filled with material of real interest and value for everyone. In this copy are general comments on fibers, how to care for furniture, rugs and clothing, then plant sources of fibers, animal sources, and synthetic sources. Processing and testing fibers close the story.

A BLANK PERIODIC TABLE FOR CHEMISTRY

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The following plan for a blank periodic table along with a discussion of its uses is offered to other chemistry teachers after it has been found to be a useful aid in teaching the periodic table and group relationships of certain elements to beginning chemistry students. A table of this type has several advantages over a complete chart. However, it is not the purpose of the writer to under-rate any of the fine prepared periodic charts on the market.

A blank chart can be constructed as shown in Figure 1 from a good grade of wallboard (of the paper type) and layed off in the conventional spaces. The size will depend upon the number of students that will observe it in the class. A chart 4′×7′ has been found to be satisfactory as it is large enough to be seen clearly by all the members of an average sized class. The information cards can be hung in the proper spaces on wooden pegs which have been glued into drilled holes on the chart. If pegs are placed in all of the spaces including the blank spaces in the row that contains only hydrogen the chart will be more useful as a drill device and for quizzes. Separate sets of cards for each major relationship to be taught will save much time in sorting cards. Different colored cards may be used to show special items.

The outstanding features of the blank chart as a teaching device include the following:

1. The periodic law can be gradually developed by starting with a series of cards containing only symbols and then following with atomic numbers. Finally the structures can be placed on the chart. This permits the building of a picture by the students by steps and without a mass of other symbols and figures on the chart. As the study continues the long periods can be developed.

2. Only elements being studied need to be on the board at a given time. This permits good visual concentration.

3. After the chart has been used to develop the periodic law it can be cleared and used for group and family study. Late in the course the families can be assembled and a comprehen-

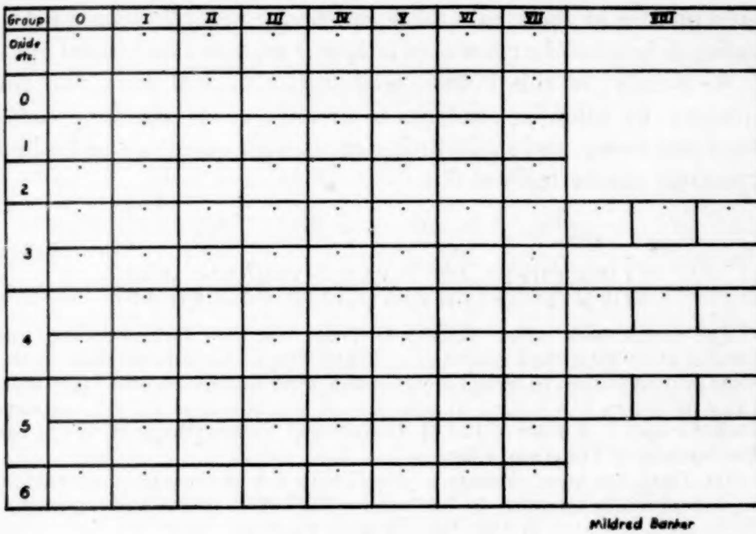

FIG. 1. Blank Periodic Chart.

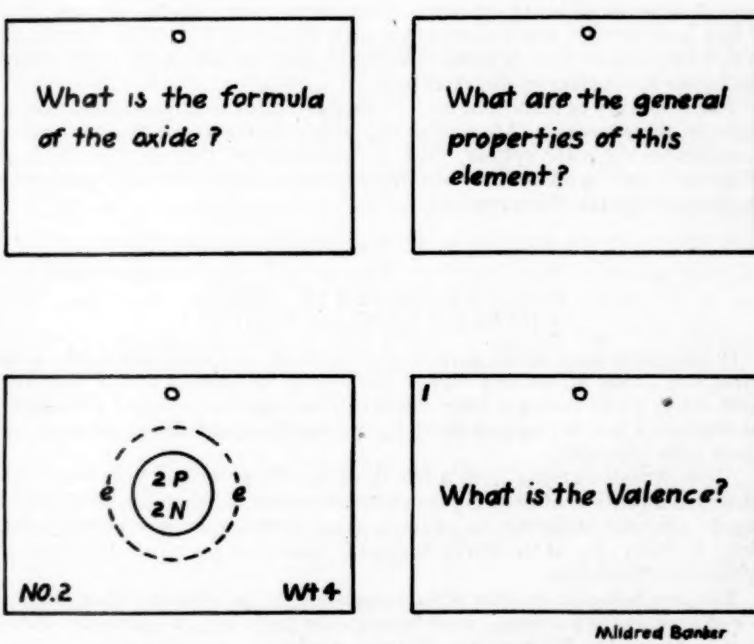

FIG. 2. Types of Cards to be Placed on the Chart.

sive picture of the whole table can be formed. *It is more interesting to help build a chart than to have it presented all worked out.*

4. A chart of this kind is well suited to drill work and for quizzes. By allowing students to arrange cards, omitting cards here and there, and substituting cards with questions unlimited exercises can be worked out.

ATMOSPHERE OF TITAN, SATURN'S MOON, FORMED AFTER SATELLITE COOLED OFF

The atmosphere which surrounds Titan, Saturn's largest moon, was formed after Titan had cooled off. Should Titan, the only satellite in the solar system known to have an observable atmosphere, become more than twice as hot as it is today, its atmosphere would escape entirely, reports Prof. Gerard P. Kuiper of the McDonald and Yerkes Observatories of the Universities of Texas and Chicago.

"If Titan has gone through a period with a high surface temperature, as is commonly assumed to be true for all bodies in the solar system," Prof. Kuiper states in the *Astrophysical Journal*, "then it follows that Titan's atmosphere was formed subsequent to that period,"

It is of special interest that, like Saturn itself, Titan's atmosphere contains methane gases that are rich in hydrogen atoms. Such gases had previously been associated with bodies having a large surface gravity.

It is highly probable, he states, that Titan was formed within Saturn's system instead of being captured from some other planet. The color of Titan is orange, in marked contrast with Saturn and its other satellites, which are yellow. This difference, however, may be due to the atmosphere acting on the surface of the satellite.

The discovery of atmospheres for other satellites would probably throw light on their creation. After studying with a spectroscope the ten largest satellites in the solar system, Prof. Kuiper suspects that of these Triton, Neptune's only satellite, may also have an atmosphere. Further study will be needed to settle the question.

BUILT-UP INLAYS IN TEETH CHANGE CHEWING HABITS

If you have your teeth corrected to get rid of malocclusion, the first meal you enjoy afterwards will cause you to be acutely aware that the new inlays make eating a little difficult. The vigorous upward movement of the lower jaw is stopped short by the newly raised dentition and you chew with difficulty.

After periods varying from a few days to several weeks, however, complete readaptation of chewing movements occurs, so that the new movements are just sufficient to effect normal pressure in jaw closure, Dr. John P. Foley, Jr., of the Psychological Corporation reports in the *Journal of General Psychology*.

The psychological picture in such instances is the same as when people, for experimental purposes, wear lenses over their eyes to turn the world upside down, Dr. Foley states. It takes a while to get accustomed to the change.

DEVELOPING AN UNDERSTANDING OF PLACE VALUE IN SECOND AND THIRD GRADE ARITHMETIC

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During the past decade, educational literature dealing with arithmetic has been placing considerable emphasis on the development of meaning of the arithmetic processes. This trend is in conformity with certain developments in the field of learning theory, so admirably outlined for arithmetic teachers by T. R. McConnell in the book, "Arithmetic in General Education."[1] H. G. Wheat, in his book, "The Psychology and Teaching of Arithmetic,"[2] places as much emphasis as any book dealing with the instruction of arithmetic on the development of the mathematical meaning of the arithmetic processes. And yet, with all this emphasis there seems to be lacking some suggestions for a concrete means of teaching the pupil in the lower elementary grades the meaning of place value. The methods suggested usually are of a semi-concrete nature. For the slow-learning pupil the semi-concrete may not be too meaningful. This article will outline a concrete means of teaching place value which has been found to be successful with pupils in the second, third, and fourth grades.

The device is, of course, a very simple one. For fifteen or twenty cents a couple of hundred tongue depressors such as used by physicians and nurses can be obtained from a drugstore. About half of the number of tongue depressors are tied up into bundles of ten. This can be done by the teacher in advance of the introductory lesson or can be done by the children. If the former method is adopted, the teacher must make certain that the children understand that there are 10 depressors in each bundle.

As a means of introduction, the teacher can tell the simpler aspects of how some American Indians counted. The fact that other peoples do not use the same number words that we do is of interest to children. Furthermore, since the children have been working with a unit on American Indians within the past year,

[1] *Arithmetic in General Education*, Sixteenth Yearbook of the National Council of Teachers of Mathematics. Ch. XI. Bureau of Publications, Teachers College, Columbia University, New York.

[2] H. G. Wheat, *The Psychology and Teaching of Arithmetic*, D. C. Heath and Company, New York. 1937.

is not hard to get them interested in how the Indian counted. For example, for five, some tribes said "fingers finished" or "one hand finished"; for ten, some tribes used "one man finished." (Some tribes used "one man finished" for twenty since they counted on their toes as well as their fingers.) For eleven, some tribes said "one man finished and one on the hand of another Indian." Twenty-three would be "two men finished and three on the hand of another Indian." Some work on the easier aspects of this method of counting is possible. The children particularly love to act out the "men finished" and the "ones on the other Indian." This is done by having children stand in front of the class or discussion group to illustrate the number. Thus: For 34, three children would stand in front of the class holding up all ten fingers to represent the "three men finished." Another child would then stand showing 4 fingers for the "four on the hand of another Indian." Be sure that the tens and the ones are in the proper order when facing the class. This will impress upon the minds of the children that in the Arabic number systems the tens are always on the left.

With some work of this kind, the children can now begin building numbers by using the tongue depressors. Ask some child to pick up 23 of the sticks (tongue depressors) at the same time writing 23 on the board. The child may respond to this in a number of ways depending upon his understanding of the situation. He may pick up two bundles (of ten) and three sticks or he may count out twenty-three sticks. If he chooses the latter, the teacher may ask if someone in the group could do this faster. The teacher, of course, is looking for a response which shows that the child recognizes that 23 is two tens and 3 ones. After the correct response has been given, the teacher asks, "How did you know how many tens to pick up?" "How many ones?" Writing a big 23 on a piece of paper and having the child place two bundles under the two (for the tens) and three ones under the three (for the three in 23) will help to fix the idea.

This work should be continued until the child recognizes that he can pick up, say 56 sticks, by picking five bundles and six sticks. In other words, recognize the place value of the five and six in 56.

To test how well the place value idea has been grasped the teacher can ask the pupil how he would write three tens and six. Some children will respond as follows: 10 10 10 6 which is a correct response. However, if the teacher solicits other re-

sponses by asking if anyone has a different way or a better way, most any group will produce the response desired, that is 36.

Over a period of time these ideas can be fixed as indicated above in a concrete fashion. Extension into semi-concrete and abstract symbolism must, of course, proceed as soon as the group has mastered the idea.[3]

Having developed an idea of place value, the child is now ready to begin adding two place numbers with carrying. (Assuming, of course, that the two place numbers without carrying have been taught.) The introduction of the process would involve a situation in which, say 23 and 39, must be added. The problem can be set up with the tongue depressors by putting the three bundles of ten and nine ones immediately under the two bundles of ten and three ones, just as it normally would be written on paper. The teacher then asks the group to find out how much 23 and 39 are. Some children will respond by counting 10, 20, 30, 40, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62. This rsponse is, of course, correct, but not the mature response to this situation. Upon asking whether anyone could count the sticks faster some child will see that it is possible to take the nine ones (in 39) and one of the three ones (in 23) thereby making a ten. Then there are six tens and 2 ones or sixty-two. With some practice working their addition exercises by means of sticks as indicated, the child has a concrete method for seeing that we organize our quantities by tens—the hundreds place is to be developed later.

Borrowing can be taught by setting up the situation so that the child has 46 sticks (4 tens and 6 ones) and asking that 19 sticks be given to someone else. The request cannot be granted until he breaks up one of the bundles of ten thereby creating 16 ones. It is now possible to take 9 ones and 1 ten from the 3 tens and 16 ones.

This method of teaching place value, carrying and borrowing has been successfully carried out in working with children in many of the rural schools of Iowa. It has its value in that it is an objective operational means of teaching place value. The idea of place value is a very important idea and should not be glossed over with the verbalizations such as "26 means 2 tens and six" unless the teacher has evidence that the words "two tens and

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[3] For suggestions on teaching the place value of number in a semi-concrete situation see: H. G. Wheat, *The Psychology and Teaching of Arithmetic*, p. 292–295. D. C. Heath and Company, New York, 1937, and *Iowa Elementary Teachers Handbook*, Vol. VIII, Arithmetic State Department of Public Instruction, Des Moines. 1944.

six" really mean something to the child. All teachers are familiar with the fact that after a certain amount of drill, the child can be made to say the proper number of tens and the proper number of ones even though the words may not be too meaningful to the child. The generalization of the method to include the hundreds place has not been carried out in this paper since it is obvious that bundles of hundreds could be made if the teacher so desires.

If it does not seem desirable to work with only one pattern, the method outlined above can be used with toothpicks, each child having his own pattern with which to work. Toothpicks are a little hard to pick up for third grade children. However, if the teacher allows sufficient time, they work about as well as the tongue depressors.

In the literature on the teaching of arithemetic there is considerable emphasis on the zero as a "place holder." Now from the historical point of view zero did come on to the scene largely as a place holder although it was probably not recongized as such at the time. However, for the present day, to place too much emphasis on the "place holder" idea for zero is probably questionable. All the integers less than 10 are place holders when used to make numbers greater than or equal to 10. The special emphasis is likely to make the teacher feel that there are some magical powers about zero when mathematically it is on the same footing as all the other integers as regards place holding. Zero does have certain properties, by definition, that other integers do not have. One of them, relating to division, should never be mentioned in the elementary school. The so-called "special properties" of zero come about as a result of its use in connection with the fundamental operations of addition, subtraction, multiplication, and division, not because of its role as a place holder.

CIVILIANS DRINK MORE MILK

Civilians have been drinking more milk and eating more meat since the war. They are now drinking between 20% and 25% more milk than they drank in the prewar days, according to the War Food Administration. During the first three months of 1944, the average American was eating meat at the average annual rate of 158 pounds, as compared to 126 pounds of meat each year in the late thirties.

SOME NOTES ON BEHAVIOR

Part Two*

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An early Greek philosopher (Socrates) said, "Know thyself." By *self* it is inferred he meant the workings of the ***mind***. Toward this end a study of the comparative anatomy of the brains of vertebrates correlated with their general intelligence and behavior is an invaluable aid.

The human body, as Darwin once said, still bears the indelible stamp of its lowly origin. The nervous system is no exception in this respect, for it is rich in the records of changes consummated in the remote past of our ancestors. A study of the comparative anatomy of the nervous system has given and is still yielding important clues to the meaning of parts which otherwise could scarcely be more than structures requiring identification.

From the dogfish to man, the vertebrate brain is based upon a fundamental ground plan consisting of five parts arranged in linear order. Beginning with the most anterior division they are: *telencephalon*, *diencephalon*, *mesencephalon*, *metencephalon*, and *mylencephalon*. Although all vertebrate brains consist of the same chief parts, the different classes vary enormously in their elaboration of detail and in the relative emphasis which they place on the different brain-regions.

Amphioxus has hardly any brain—the front part of its spinal cord barely differs from the rest. This is because it has few special sense organs on its head; it lacks eyes, ears, and a nose. Moreover, it is jawless, feeding by means of an endostyle which is automatic and independent of nervous control.

With continued movement with one end anterior there arose a need of sensory structures to inform the animal of the immediate surroundings that it entered. Later a jawed mouth came. Thus arose the need of a brian to control the jaws and to receive the reports from the sense organs. As these sense organs, the nose and eyes in particular, became better and better developed, the brain became more and more dominant over the rest of the

* Part One appeared in the March issue of SCHOOL SCIENCE AND MATHEMATICS. Part Three will appear in a later issue.

central nervous system. It grew to be the best informed part of the body—and gradually all other parts became subordinate to it. They sent their sense information up to this main center to receive from it their instructions.

As in all vertebrate systems, there are two aspects of the evolution of these parts. One aspect is the emphasis any particular vertebrate may place upon certain of these parts to meet its own peculiar needs. In particular, I should like to emphasize the marked effect that two of the organs of special sense, namely, the eyes and nose, had upon the relative development of brain parts. For example, the brain of the dogfish is primarily a "*smell brain*," the sense of smell dominating with a corresponding emphasis on the development of parts concerned with olfactory reflexes, i.e., olfactory bulbs, olfactory lobes, etc., whereas the brain of a trout is primarily an "*eye brain*" with a corresponding emphasis on parts of the brain concerned with sight reflexes.

The other aspect is the progressive trend toward the development and elaboration of the more anterior parts of the brain and their dominance over the phylogenetically older more posterior portions. This is obvious when we compare the relative proportion of the telencephalon of a teleost and man. In man, the cerebral hemispheres constitute seventy per cent of the weight of the entire nervous system, whereas in teleosts somewhat less than two per cent is thus involved.

Using the dogfish brain as an example of a primitive vertebrate brain, we shall mention the functions and relative development of the various parts and correlate these with the behavior of this form, and mention differences from the brains of other fish.

Functionally and anatomically the *medulla oblongata* is very similar to the *spinal cord*, with which it is directly continuous. Both are concerned with *automatic reflex activity*, making a stereotyped response to any given stimulus, such as a knee jerk compared with heart action of a running man. The chief difference, however, concerns the structures involved. The spinal cord is primarily, though not entirely, concerned with somatic reflexes as striated muscle; the medulla is fundamentally concerned with splanchnic autonomic reflexes like the heart rate, the rate and depth of respiration, and gastro-intestinal activities, such as automatic taste, swallowing, secretory and peristaltic reflex. In addition both are concerned with transmitting

stimuli up and down the neuroaxis along certain definite stereotyped nerve pathways or "tracts" as they are called.

The medulla, lying dorsal to the anterior end of the gastro-intestinal tract, innervates this structure. Primitively, respiration is carried on by means of gills which because they are developmentally part of the anterior end of the gastro-intestinal tract have the same nerve connections, namely, with the medulla, a connection which was transferred to the lungs when they were evolved. As a result of the association of the gill mechanism with the process of aeration of the blood, a part of the circulatory system, the heart and aortic arches traversing the gills, likewise have medulla connections. Because of these primitive relations, the gastro-intestinal tract, gills, and circulation all have connections with the medulla.

In addition to visceral regulation the medulla has assumed the function of reflexly maintaining the balance of the body. This is accomplished by means of its innervation of the lateral-line system and its derivatives, the semi-circular canals whose function is dynamic equilibrium, and the utriculus and sacculus whose functions are static equilibrium. Just what the lateral-line organs do is by no means clear. Some writers believe that they detect currents, others that they are sensitive to pressure, others again that they respond to slow pulsing vibrations of the surrounding medium. Perhaps the most plausible suggestion is that they feel the force with which the fish's flanks press against the water. We may assume that they have a delicate sense of contact between themselves and the water, responsive to every slight variation in intensity, and that this sense is one of the regular pervading elements in the background of the fish's mental life. The lateral-line organs linger on in aquatic amphibians, in tadpoles, and many newts, but disappear in the definitely terrestrial frogs and toads. To summarize, we may conclude that automatic reflex centers for hearing, balance, respiration, cardiac action, and gastro-intestinal activities are located in the medulla.

No great changes have been made in these primitive automatic connections from sharks to man. They function today as they did some half billion years ago, as automatic reflex centers, giving stereotyped responses to any given stimulus.

From the earliest stages of vertebrate history the function of the *cerebellum* has been to *coordinate muscular movements.* This function has been handed down to man unchanged.

The cerebellum is indirectly connected with all the receptors, eyes, ears, nose, taste, and general sensations and affectors, muscles of the body. This indirect connection results in slower reflex response, but a greater degree of correlation is attained between the various incoming stimuli and the motor impulses. We would expect to find, then, in sluggish vertebrates like cyclostomes, amphibia, and reptiles, poorly developed cerebelli, and we do. On the other hand, active forms like the dogfish (and by the way, the cousins of the dogfish, the sluggish skates and rays have small cerebelli) teleost, birds, and particularly mammals have large, well developed cerebelli.

Up to now we have spoken of the brain as a structure designed to convert sensory stimuli into simple and immediate stereotyped reflexes adapted to meet the routine requirements of the body. Such was, no doubt, the earliest vertebrate brain. No provisions had been made for a great variety of responses which could be made individually to a variety of different situations. Their behavior would have been strictly limited to stereotyped taxis. However, even in cyclostomes, an organ was developed which was capable of forming important *correlations of sensory stimuli* without being burdened with the duties of synergistic control or with the routine control of muscles, heart, respiration, etc. Such a center was the *mesencephalon.*

The visual, auditory, and bodily sensations had their endings in the mesencephalon and were correlated here to produce the most effectual motor responses to be made such as "Shall I attack or run away?" The command originating in the mesencephalon was carried out reflexly by the lower centers. Some form of consciousness probably accompanied such reactions, and, if it did, its seat was in part in the mesencepahalon. We see then that some of the functions carried out by the cerebral hemispheres of higher forms resided in the mesencephalon of primitive vertebrates such as fish and amphibia. To summarize, we may say that in the lower vertebrates, the mesencephalon was the controlling part of the brain, receiving information from sense organs, making correlations and sending out orders to be automatically carried out by lower reflex centers. In addition it is probable that some form of consciousness attended these activities and hence in part had its seat in the mesencephalon.

The higher sensory correlations resulting in a *sense of well-being* are located in the thalamus. The thalamus is an important

part of the *diencephalon*. It is here that the organism as a whole takes stock of itself—"Am I feeling good, bad, or only fair to meddlin'?"

It is probably this part of the nervous system in the lower vertebrates that is designed for an extremely high degree of sensory correlations. All sensations are assembled here, and the proper correlation of these sensations results in the general sense of well being, of body tone—"All goes well or does not go well." Excessive stimulation of skin due to much light, impulses of an empty gastro-intestinal tract, hunger, distension of ovaries or testes with ripe sexual products all dixturb the sense of well-being and result in appropriate reactions to restore the normal state. Such protective reactions save the animal from starvation, and from hostile elements, and also perpetuate the species by appropriate sexual activity. The reactions are phyletically conditioned responses, instinctive reactions, growing out of the necessity of maintaining a sense of well-being.

Hunger, for example, produces a decided change in the sense of well-being which depends upon impulses from the stomach. This results in the carrying out of the necessary reactions which have proven successful in the history of the ancestral forms in the securing of food. Likewise sexual emotions, primitive emotions of fear, and rage arise from combinations of somatic and splanchnic impulses, and appropriate reactions are made to preserve or perpetuate, as the case may be, the species. The behavior reactions, then, have been phyletically conditioned and are of a primitive sort evolved in the hard school of animal savagery and preservation. Those of you who saw the "Roaring Twenties" with the ruthless gangland warfare have an idea what is meant,—men with their thalami operating at full blast and dominating their behavior.

The diencephalon is involved in the following:

1. General sense of well-being or body tone.
2. Instinctive behavior.
3. Primitive emotions: seat of primitive fears, rage, sex emotions, etc.
4. Probably the main seat of consciousness, for all other suprasegmental structures have been removed without destroying the state of awareness.

In addition to these functions the diencephalon is concerned with activities often called the *essentials of life* such as control

of water metabolism, temperature control, sleep, etc. Certain other structures are associated with the diencephalon, such as the hypophysis, the pineal gland, and sacculus vaculosis. The first is hormonal, the second a structure of doubtful function, the last a mechanism associated with the sense of pressure in fish.

The primitive telencephalon, or cerebrum, is made up of a roof or archipallium which receives sensations from only one source, the olfactory lobes located at the anterior end of the telencephalon. In the floor and ventral side walls of the telencephalon is the corpus striatum, a nerve center, which receives orders from the smell brain and thalamus and carries them out.

The essential point to be emphasized is that the sense of smell dominates the behavior of the dogfish with regard to external stimuli. The archipallium and olfactory lobes are given over entirely to the *sense of smell* and therefore dominate the activity of the *telencephalon.*

In the lower vertebrates one sense organ or another may dominate the activity and development of the various parts of the brain.

For example, the trout is predominantly a visual organism: its optic lobes are very large, and, although other sensations are considered here, the optic center is chairman of the conference, so to speak, and has the final say. Decisions are therefore mainly influenced by the eyes. As any keen angler will tell you, the trout has a critical eye for the appearance of the lure. Usually, it will strike at the pattern which most closely simulates the kind of insect that is most prominent at that time. But fly fishing for a dogfish would be an entirely different game —instead of tinsel and jay feathers the bait would be flavored with judiciously chosen scents.

The carp, on the other hand, with well-developed taste buds in his mouth and on his back and body is an incorrigible epicure —with well developed taste centers in the brain dominating its behavior. If you quietly drop a bit of meat close to the flanks of a carp or catfish, the animal will twist round and snap it up at once, by virtue of its diffuse powers of taste.

Professor Herrick, noted neurologist, has often made the statement, "Show me the brain of a vertebrate and I'll tell you the *habit, abilities, and general ingelligence* of its possessor."

With this in mind what can we say about the fish, the amphibia, the reptiles, the birds, and mammals?

Fish

Fish live in a curiously circumscribed world. They can see only a short distance, and sound waves travel poorly. Even smell is a relatively short distance receptor. The majority of fish sniff at things and thus find their prey. In the matter of other senses, touch, pressure, pain, and so on, fish are probably very much as we are, save for an outstanding exception. Even if they are sensitive to temperature they can seldom experience such sensations as the sudden warmth of a ray of sunshine or the chill of a breeeze, because of the very uniform temperature of the water in which they live.

As for the intelligence of fish, most people who have studied them carefully are agreed that they are stupid and learn but slowly. Triplett kept a pike in an aquarium with a number of smaller fish, but he separated the two by means of a glass plate. Soon the pike learned that to leap at the other fish was to get a sharp blow on the nose—although it could not possibly understand the reason, for the glass plate was perfectly invisible. When the glass plate was removed, the pike swam around with the other fish, but it never made any attempt to seize them.

Amphibia

The frog is a sluggish creature lying on its belly firmly supported by the ground. It needs little of the highly developed balancing and consequently coordinating ability of the fish who lives in a fluid medium. Correlated with this ecological difference is a morphological difference of the cerebelli of fish and amphibians. The amphibians have a smaller, less highly developed cerebellum than the fish.

The medulla, or brain stem, functions much the same in all vertebrates, for all have the same type of automatic reflex activity underlying their behavior.

The telencephalon in the frog is about the same size as that in the fish and occupies a somewhat intermediate stage in importance between the two extremes presented by the dogfish and the trout. Although the archipallium is given over entirely to smell, this part of the brain is not as influential in determining behavior as is the mesencephalon.

The cerebral hemispheres are largely dominated by the sense of smell; archipallium, entirely so. The thalamus is making stronger representation to the corpus striatum than it is in the fish.

The great development of the optic lobes would seem to indicate that the eyes are the important organs in the determination of behavior in frogs. Anyone who has observed a frog, blinking in the sun, gets the impression that most of the appreciation of the external environment comes through the eyes.

Even though the eye of the frog has undergone considerable development, the centers of appreciation in the cerebrum have not yet appeared. If an object such as a fly does not move, frogs do not recognize it as living. A frog will as readily snap at a pencil in motion as it will a moving worm.

Although in an environment offering unlimited possibilities for sense organ development and consequent brain development, the amphibia have made only the crudest beginnings along these lines. Compared to the next class, the reptiles, the amphibia are morons.

In some respects the amphibians are less well equipped mentally than the majority of modern fish. From an evolutionary point of view, they represent a stage when the attention of the stock was focused on those alterations which made land-life possible: on the shift from gill-breathing to lung-breathing with its consequent revision of the whole layout of the throat, the heart, and the main arteries. *It may be their very mental inadequacy which, by putting them at a disadvantage, compared with their competitors in the waters, forced them to make these adjustments and come on land.*

Reptiles

But when the vertebrate body had been properly fitted for a terrestrial life, when the dry impermeable skin and the protected egg, which characterize all vertebrates above the amphibian level, had been evolved, there arose a new era of intense competition on land. Terra firma ceased to be a refuge for the witless, and except for a few out-of-the-way corners, became a hard school in which only the nimble and vigorous, or the exceptionally well protected, could hope to survive.

Inasmuch as the functions and structure of the brain stem are essentially similar in the vertebrate clan, relegated as it is to reflex behavior, it becomes necessary now to concentrate our attention on the telencephalon, which is destined to assume in birds and mammals the greatest development of any part of the nervous system and correspondingly becomes the dominant controlling center of the brain.

The most essential difference between the brains of amphibians and reptiles is the appearance in the latter of a new way of arranging grey matter in the walls of the cerebral hemisphere. In fish the grey matter is massed next to the inner cavity of the cerebrum. In amphibians the grey matter or nerve cells are evenly distributed throughout the walls of the cerebral hemispheres. In reptiles, birds, and mammals the grey matter appears as thin sheets in the roof of the hemispheres, separated from the ventricles by layers of white matter of varying thickness. These grey sheets lie right at the surface of the cerebral hemispheres, and they are therefore spoken of as the cerebral cortex. This is a complete reversal of the condition present in fishes. The cerebral cortex is found in reptiles, birds, and mammals. Its appearance is a tremendous stride away from the rigid automatic behavior towards intelligent behavior. But curiously enough, its possibilities are only fully exploited in the mammals.

Another essential difference between amphibia and reptiles is the appearance of a new area in the cerebral pallium, the neopallium, a portion of the cerebral pallium for the first time not connected with the sense of small but free to make connections with other sensory impressions. The neopallium is destined to become the dominating portion of the brain of mammals, the seat of all involuntary activities and higher psychic phenomena.

The brain in land-living vertebrates evolved along two different contrasting lines. One leads through the dinosaur-like reptiles to birds; the other leads to the mammals. In the bird development, the principal evolutionary change was the enlargement and complication of the corpus striatum. That part of the brain may be said to culminate in birds. Nowhere else is it so elaborately organized; nowhere else does it outweigh the other parts so markedly in size. But while the floor of the forebrain was thus undergoing improvement, the roof remained thin and the cortex small. In the mammal direction, the opposite development occurred. The corpus striatum did, indeed, undergo a certain amount of revision and reconstruction, but the main progressive change was the enlargement and elaboration of the cerebral cortex. As we shall see in a moment, this difference is the cause of a very striking contrast between the behavior of birds and that of mammals.

How does this structural divergence between the brains of reptiles and birds, on the one hand, and mammals, on the other, reflect itself in their behavior?

Aves

Birds can learn. Anyone having seen performing pigeons knows that their brains can hold one or two tricks. But preforming pigeons generally come on the stage in great flocks, for the number of tricks that any single bird can do is much less than that of a good performing mammal. Indeed, the learning ability of a bird is hardly greater than that of a reptile, and certainly less than that of a rat. Where birds excel is in the performance of elaborate but purely instinctive acts. An incubator-hatched bird will build its nest as neatly and select its material as carefully as any other member of its species; at the first appropriate occasion it will go through the routine of courtship with perfect correctness. In a word, the great development of the corpus striatum in birds means a tremendous endowment of inherited instinct, rivalling that of insects in its complexity and far exceeding our own, but without any notable individual intelligence or adaptability to modulate it.

An admirable example of blindly mechanical instinct is afforded by the so-called thermometer bird, or bush turkey, of the Solomon Islands. The eggs of this species are laid in heaps of mixed plant material and sand, and incubated for some six weeks by the heat of the rotting vegetable matter. Moreover, they all lie with the blunt end upwards. The chicks hatch out from this blunt end, and their feathers all point stiffly backwards; so by wiggling and struggling in the heat they can force their way upwards. As soon as they get out they shake themselves, and then dash into the shadow of the nearby undergrowth. Here we have an admirable adaptive chain of reactions, or chain reflexes. But if you take a chick which has just liberated itself and dig it into the heap again, it is quite incapable of coming out once more, but stays there struggling ineffectively until it dies. Its movements are now of the wrong kind. The reactions follow one another automatically, a mechanical chain of reflexes, and the creature can no more go back to the beginning and start again than can the cocoon-spinning caterpillars repeat the formation of a new cocoon when artificially removed from a cocoon.

But meanwhile the other vertebrate line that leads up to the mammals took a somewhat different direction.

Mammals

The outstanding difference between mammals and other vertebrates lies in the adaptability of the former. They can remem-

ber more surely and learn more quickly. Moreover, they tackle a new problem more competently. Suppose you put a fish or a frog or a reptile in some experimentally devised strange situation, such as a maze, in which the animal has to follow a definite path if it is to reach a comfortable nest and food, while other possible paths lead nowhere. It will simply muddle about among the unfamiliar surroundings until it gets to the goal. Put the animal in again, and it will muddle its way through again; but gradually as the lessons are repeated, it will get less and less dilatory and make with more and more directness and certainty for the goal. This kind of learning is simply trial-and-error, like the random trial-and-error of a Paramecium, but with a certain element of memory attached to it.

Yerkes, an American investigator, devised a maze for some tortoises on which he was experimenting. Part of the path to the comfortable nest, to which they were supposed to find their way, led down a steep inclined plane. It so happened that one of the tortoises began poking aimlessly about and tumbled off the side of the sloping plane. Surprisingly, it found itself near a pleasant nest. Thereafter this animal invariably solved the problem by repeating his accident—by making for the edge of the plane, deliberately tumbling over, and then calmly walking into the nest.

To a large extent, all animals learn in this way, by muddling about more or less at random, and by remembering which activities were followed by pleasing results and which by harmful ones. But mammals do it very much more quickly and less laboriously than other vertebrates. Moreover, at least in the higher mammals, there often appears a somewhat different kind of solution when the animal comes up against anything new. The creature supplements this profiting by chance events by means of inner resources of its own. It stops to think; then it "gets an idea"; then it tries the new idea to see whether it will work.

The evolution of the mammalian brain is largely a matter of the improvement and development of the cerebral cortex and the neopallium.

First, we note there is a tremendous enlargement of the telencephalon, which in man has reached the stage where it has grown over and obscured from view most of the brain stem.

Second, the neopallium becomes progressively larger and the archipallium progressively smaller.

Third, in connection with the enlargement of the neopallium,

there is an increase of voluntary activities and higher psychic phenomena. It is here that centers of speech, hearing, vision, thought, and associated memory reside.

With the development of good eyes and skillful hands there was evolved a tremendous increase in the ability to explore and perceive the outside world. This increased perceptibility is reflected in the enlargement and development of areas in the cortex assigned to these duties, and in addition new areas not allotted to any particular function appeared. These newer areas, free from the immediate reception of various sensory stimuli, are in a position to correlate and associate the sensation-association areas. In these areas the thinking process is consummated.

Thus the mammalian brain has evolved. The primitive amphibians are spawned in great multitudes; they are stupid, inadaptable creatures, but out of the thousands thus launched a few survive to carry on the species. The higher mammals reproduce altogether more sparingly, but they are better able to take care of themselves and to survive unusual combinations of circumstances. *A large share of the responsibility for carrying on the race has been handed from the genital organs to the brain, from the proliferating germ-plasm to the thinking individual soma.*

Mental hygiene may be thought of as that field of human knowledge which aims to direct the development of the individual into a more or less stable state of mental and behavioral normalcy. If one's behavioral pattern is such that inherited emotional drives far out-balance cerebral inhibitions, the individual may develop into a gangster, a rapist, or just the black sheep of the family. On the other hand, should cerebral inhibitions play a controlling or dominant role, the result may be some form of psychoneuroses. Normalcy in human society requires a balance between inherited emotional drives, which are a form of instinctive behavior controlled largely by the thalamus and corpus striatum, and cerebral inhibitions which are a form of intelligent behavior controlled largely by the cerebral cortex and neopallium. In one sense education is training directed to modify activity of the thalamus by cerebral inhibitions.

The Common School is the greatest discovery ever made by man.—HORACE MANN.

MATHEMATICS FOR ALL AND THE DOUBLE TRACK PLAN*

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In approaching this subject, I wish to be thought of not as an educationist nor as a former teacher of mathematics alone, but as one who has for a quarter century been active and maintained a keen interest in both areas. In discussing the future of mathematics in the high schools, two kinds of approaches have been commonly employed—one of these has been far more popular and in all likelihood less useful. It is what may be called the "Pollyanna" or "rabble rouser" approach. The author or speaker makes the strongest case he can for teaching mathematics in the schools and refuses to recognize any of the dangers or any of the needs for improvement or reorganization of the content. The second approach is much more hazardous, though probably much more serviceable. It may be called the "Prophet Jeremiah" or "Dutch Uncle" approach. In this approach the speaker or author takes for granted that mathematics should be taught to more and more people. He addresses himself to the dangers in the present situation and dwells greatly upon the weaknesses and upon the possibilities of improvement of the present offerings. I would have very little interest in approaching this discussion along the first of these two lines.

I wish to make two principal points:

1. Everyone should study mathematics in high school and should have at least a year of mathematics beyond the tenth grade, and in all, not less than two years of high school mathematics.

2. The conditions of the times clearly call for two sequences of mathematics in the high school (a) one for the growing mass of high school students, a sequence to give them the mathematical information, skills and attitudes necessary to meet problems of everyday life in home, business, recreation, vocation, citizenship and health and (b) a sequence for the specialist—the student of more than average ability in mathematics—the future engineer or college student in some phase of physical science.

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* An address to the high school section of teachers of Mathematics of the Central Association of Teachers of Science and Mathematics, Chicago, Nov. 28, 1944.

One Thing We Learned from the War

From our experiences during the war we have been stimulated to think more carefully about the need for training in mathematics. I served as a member of the committee to select students for the Navy V-12 program and from that experience learned a great deal about the needs of the Navy and about the background of young people today in mathematics. I should like first of all to call your attention to the fact that the officers of the Navy who wrote or spoke on these matters emphasized:

1. that a considerable minority of young men entering the Navy should be well trained in mathematics beyond algebra and geometry, and
2. that the great majority should be thoroughly trained in arithmetic, and elementary algebra and preferably also in elementary trigonometry. In the refresher or pre-induction course of study in mathematics planned by the Navy it is recommended that *40% of the time be given to reviewing and extending skills in arithmetic.*

How Much Arithmetic in the High School Mathematics Courses?

Perhaps those of us who are inclined not to take seriously the suggestion that considerable attention be given to arithmetic in the high school should remember that at one time arithmetic was taught at Harvard University and other leading institutions of higher education, and that for a long time it was a most respectable secondary school subject being required for entrance to colleges and universities. It is easy to understand the pride of one, who having majored in mathematics in college, and how that pride might mislead one to the conclusion that he or she is above teaching arithmetic, particularly when one knows what difficulty many mathematics majors have in solving many of the problems in arithmetic and how lacking in background they are with respect to the areas of application of arithmetic. It is also not impossible to understand how that pride may stand in the way of the welfare of mathematics and of secondary education.

There are several reasons, however, why we must at this time consider the matter more carefully. In the first place there is a growing and articulate criticism of the graduates of our high schools because of their very inadequate mastery of arithmetic. In addition, the postponement in grade placement of more diffi-

cult topics in arithmetic in the elementary school has made it impossible for elementary school teachers to complete the study of arithmetic before the ninth grade. It is futile to suspect that this postponement policy will be reversed. Teachers in the elementary schools have come to believe fervently in the futility of attempting to crowd learning of mathematics too rapidly. On every hand, we are discovering so many individuals who not only did not master arithmetic in the elementary school but who as a result of this crowding process have developed antipathy towards mathematics and a lack of confidence in themselves which is almost psychopathic.

If this postponement movement should result in deferment of systematic instruction in formal algebra to the tenth grade, then we must consider favorably beginning of algebra at that time and the teaching of a great deal of arithmetic in the senior high school. Such a plan of postponement would not be without important advantages for instruction in algebra, begun at a later age and taught to a more select group who would have less time to forget it before entering college. The way of the algebra teacher of today, like that of the transgressor, is hard, expending as he or she does, major efforts and worries in a futile attempt to make mathematical silk (rayon) purses out of sows' ears, to the shameful neglect of the able student.

More Students or Fewer to Study Mathematics in High School?

The present increasing popular interest in mathematics is likely to be ephemeral. The public is fickle and its interests shift quickly. We must "strike while the iron is hot." We should keep our customers coming by giving them the type of training most valuable to them and most completely in harmony with the needs of the time and the future.

Enrollments in mathematics in the high school school have diminished steadily for forty years, most probably because our courses, as offered, have not been suited to the needs of the masses in modern times. We have in our actions said, "Here is our course for the specialist. Take it or leave it. If you have no practical need for it be satisfied that it has disciplinary and general mental training." Below are statistics demonstrating the failure of that policy to be convincing as indicated by the gradual disappearance of high school mathematics from the programs of the majority of high school students. The figures in the state-

ments are percentages of all students in the high school enrolled in mathematics classes:

	All Algebra	Plane and Solid Geom.	All Math.
1900	56.3	27	91
1922	40	23	64
1928	35	19	57
1934	25	13	44
1940 (Estimated)	22	10	36

	Elem. Algebra	Plane Geom.
1928	27	18
1934	19	12
1940 (Estimated)	15	9

We should bear in mind that these decreases have taken place in spite of the continued requirement of mathematics for college entrance—a crutch which has, I fear, caused us to take it too easy and to worry too little about the future of secondary school mathematics. There are indications that the prop of college entrance requirements may be gradually pulled from under us. Already there is a movement under way to discontinue the requirement of mathematics for general college entrance. Not only do many teachers colleges no longer have such a specific requirement but entrance requirements have been revised so that students may enter without credit in either algebra or geometry, in a great many colleges and in a number of our larger universities including such institutions as the University of Michigan, University of Iowa, University of Wisconsin, University of Illinois and Stanford University. Repeated careful investigations have shown that in colleges of arts and science, schools of music and schools of business administration, students who have not presented, for entrance, credits in high school mathematics make as high scholastic average as do students of the same general intelligence who have had two or more years of high school mathematics.

Everyday Needs Becoming More and More Mathematical

It defies explanation and is contrary to all logic that as the world becomes more mathematical, fewer students in high school take courses in mathematics. There is a screw loose somewhere—an Ethiopian in the proverbial woodpile. If time were available and if it were not carrying "coals to Newcastle," I would like to help you picture in your minds this afternoon just

how mathematical a world we have become and just how much more mathematical we are becoming, how almost every hour in every day a great majority of us have some thought partaking of a quantitative nature if not actually involving some computation and mathematical problem. I want to point out just a few of the elements in this trend:

1. The home of today is no longer a place of self-sufficiency. It has become and is becoming more and more a place of purchasing rather than producing. Even in our processing we have become more mathematical.

2. In the home of today financial matters have become more definitely mathematical as exemplified by our greater participation in insurance, annuities, budgets, to say nothing of the mathematics involved in all of the various scientific gadgets which make work easier in the home, or the mathematics involved in home decoration, or the mathematics relative to installment buying, borrowing, increased variety, amount and complexity of taxes.

3. Matters of health have become mathematical, what with our thinking in terms of calories, weights, etc.

4. Our methods of transportation not only are employed much more frequently but involve much more mathematical thinking, particularly as a result of the replacement of horse power in the flesh by horse-power in the combustion engine, with all its gauges and instruments and related calculations. We never thought in terms of how many miles to the bale of hay or bushel of corn, but we must think in terms of how many miles to the gallon. We must think also in terms of transportation expenses on the bus, in the airplane, by trucks, parcel post, etc., and be able to compute the costs of owning and operating an automobile.

5. In our plan of vocational activities, measurements play a much more prominent part both in original construction and in repair. This is particularly true in those vocations having to do with building construction and with gasoline engines and related machines of one kind or another.

6. To be an intelligent citizen in these days one must do considerable thinking in mathematical terms—in terms of magnitude and ratios—about the implications of national expenditures and budgets far beyond the capacity of the untrained mind to comprehend.

As Fibber McGee says these days "Pardon the expression"

but what a cockeyed world this is in which which mathematics plays an ever increasing part and mathematics education is constantly diminishing. At this point, let us consider what may be done about the matter. As possibilities I wish to present some proposals which no doubt would be modified this way or that if adopted in individual schools and which should go far to bridge the gap between the needs of today and our traditional school program.

Track One of a Double Track Plan—For the Masses

I should like to have you think about a double track plan—one for the masses and one for the specialist. Track one might have several forms. For example it might well take the form of requiring a semester of mathematics and perhaps a semester of science in every year through the high school, thus assuring that mathematics is not dismissed from the thinking of any student at the end of the eighth grade or at the end of the tenth grade and that he may grow up in an atmosphere which is somewhat mathematical and have his instruction developed along with his maturity and his expanding experience and interests.

This required science and mathematics would naturally involve much arithmetic and many of its difficult and complicated applications to the problems of life in all its various aspects. It would involve a great deal of what is now thought of as first year algebra, emphasizing as it would thorough training in the use of formula and the simple equation, graphical representation and negative quantities.

An alternative plan for Track One would involve a required year of general mathematics either in the ninth or tenth grade and again another required year in the eleventh or twelfth grade. This plan has one practical advantage in that there are textbooks already available for general mathematics in grades nine and ten and in general mathematics in grades eleven and twelve.

I am not inclined to take very seriously the claims of those who believe that we should have specialized courses in mathematics such as shop mathematics, business arithmetic, or agricultural mathematics. It is not possible to offer several such courses in small schools and the Track One general courses should serve almost equally well for vocational purposes and much better for various aspects of general education.

"Senior General Mathematics"

Already a rapidly increasing and very considerable number of schools are offering the course in ***Senior Mathematics***, some only as a refresher course, but an increasing number as something more than that—a course in the skills and applications of arithmetic, simple algebra, geometry without proofs, and simple trigonometry. In a considerable number of schools, there is taught the use of the slide rule and of logarithms on both tracks but particularly on Track One. We should give considerable attention to types of outcomes other than mere calculation outcomes involving such as (a) ability to read carefully and think logically about mathematical problems (b) certain attitudes and habits of mind such as critical attitude relative to proof, habits of thinking about cause and effect and other interrelationships, (c) importance of developing a wholesome attitude toward mathematics and of avoiding an unhealthy attitude towards it or toward the school as a result of unfortunate experiences in mathematics classes. To those who would say that these types of outcomes smack of mental discipline, it may be said that indeed they do—of discipline of a practical and attainable nature and that such discipline should also include such things as habits of checking, the use of approximate estimation and rough or rounded calculations.

In Track One, much more attention should be given to ***mental*** calculations, realizing that in a fairly considerable percentage of our needs for mathematics, we must make our decisions without the use of pencil and paper even though our calculations are only approximate.

Track Two for the Specialist

Track Two is a familiar one to most of us. It is a well-known track, one which has been employed for generations. It is the track for the specialist, the prospective engineer, scientist, or teacher of mathematics, and for those who wish to enter a college or university which still requires formal algebra and deductive geometry. This well-worn track is in need of straightening on the curves, new ballast, and streamlining. In most schools it probably should begin in the tenth grade with a thorough one-year course in formal algebra. In the eleventh grade, plane and solid geometry would constitute the work of the year and in the twelfth grade more algebra, more arithmetic and ele-

mentary trigonometry, the time being divided approximately equally.

In larger schools this track might well begin with a year's course in algebra in the ninth grade, with plane geometry in the tenth grade, algebra and trigonometry in the eleventh grade, and algebra and solid geometry in the twelfth. In such schools, however, admissions to the algebra class in the ninth grade should be given freely only to the student who is at least in the upper half of the class with respect to ability to do mathematics, probably only to those in the upper one-third. Those in the middle third might well be advised to postpone algebra until the tenth grade, and those in the lower third encouraged to avoid disaster. The teaching of plane and solid geometry in one year need not be apologized for. In the smaller schools students wishing to take solid geometry are not numerous enough to warrant giving it. In any school devoting one and one-half years to instruction in geometry there is a disproportionate emphasis on geometry as compared to algebra in view of its practical advantages, even may I say for those who will be engineers or scientists.

In those schools in which a rigorous instruction in formal algebra is begun not before the tenth grade, a course in general mathematics should probably be offered and might well be common to both Track One and Track Two. It might be, however that where this plan is followed, two kinds of general mathematics should be offered: one for the students of lower ability and one for the students of greater ability, as far as mathematics is concerned, and the courses of study appropriately differentiated. In any case, the course in general mathematics in the ninth grade should not be, as it is in some schools, a diluted or camouflaged course in albegra. It should contain considerable arithmetic, some algebra, and without question, some geometry, the algebra emphasizing skill in the use of formulas, simple equations, and graphs, and the geometry emphasizing construction scale drawings, and indirect measurement.

Will the Double Track Plan Become Widespread?

To sum up thus far, the Track Two kind of mathematics, important as it is, is not suited to the needs of the majority of high school students. We can no longer stave off the trend against us or dodge our responsibility with vague phrases about mental discipline and training the mind. Society demands that,

but also more. Not only do the needs of those in the armed forces, but also the needs of the average person, call for a disciplined mind with specific abilities to meet specific situations calling for mathematical thinking. Disciplinary values only are not enough. People are beginning to feel rather generally that other subjects probably have equivalent disciplinary values.

Not only will the percentage of students in high school studying the Track Two variety of mathematics continue to decrease but just prior to the war, in several states, perhaps many, the actual numbers decreased, New York and California for example. Even if we are not interested in numbers, we can hardly turn a deaf ear to the demands of the times and what is more to the point administrators looking for teachers of mathematics will find that after the war those who can teach Track Two courses will be very plentiful and easy to get. Those who can teach Track One courses will not be sufficiently numerous to meet the demand and will therefore be most sought after.

What stands in the way of our making the adjustment that seems indicated by recent developments and current needs. It is interesting to survey these:

1. Just plain tradition, inertia—lead in our mental boots.
2. The present abilities and attitudes of teachers. Many prefer the old way, the easy, simple way of teaching the regular mathematics out of the textbook, justifying our procedure and quieting our consciences with vague and wild theories about discipline and college preparation.
3. Many teachers are creatures of the classroom, who know little of the application of mathematics to life and are not at home in discussing such things with youngsters.
4. Many teachers either do not have, or fear they do not have, sufficient imagination, ingenuity and scholarship to organize the new courses and to teach them.
5. The conservatism of parents who apparently would like to keep the schools just as they were when they attended them and who are befuddled by theories of general mental training.
6. A considerable but decreasing number of administrators who prefer to be among the last to take up the new.
7. Suitable textbooks have not been available. This limitation is rapidly disintegrating. We now have several excellent three book series in junior high school mathematics enabling us to give an integrated sequence through the ninth grade, previous to the study of formal algebra. Within the past year several excellent books for senior mathematics have been published. Already a few textbooks combining plane and solid geometry in a one year course, have made their appearance and no doubt more such books will appear.
8. Stubbornly decreasing ignorance and misinformation about how to prepare students for college and a reliance upon the taking of certain subjects rather than upon the methods proven to be more effective in preparing students to do college work.
9. The mistaken, though genuine and sincere, attitude of a considerable

number of college professors of mathematics, who have had no experience in thinking in terms of mathematical needs of other than engineering students and majors in mathematics and physics. These men and women usually are quite unfamiliar with the high school population of today and have given little careful thought to the problem of what really are their mathematical needs. I am not one of those who feel that the college professor of mathematics is motivated largely by fear that if the newer trends in high school mathematics are adopted, teachers will have little need for courses in analytical geometry, calculus and differential equations.

Suggestions for Improving Teaching of Mathematics

I would like to conclude with a few statements about the nature of instruction particularly important for "the other half" or the other two-thirds or three-fourths who will follow Track One. In general, they possess certain identifying characteristics and there are important implications of these:

(1) They have been none too good in arithmetic. They are afraid of mathematics and dislike it. They require much patience and careful handling.

(2) They do not grasp abstractions readily. They need much concreteness, visual education of all sorts, application problems, drill and much activity on their own part.

(3) They acquire their vocabulary slowly and with difficulty. Careful definition and explanation and exemplification of all new terms is called for.

(4) They need as far as possible to develop their own rules, or at least to assist the instructor in developing them inductively wherever possible, and to have those rules made meaningful immediately by clear, practical applications.

(5) They respond well to personalization of problems, that is, a study of problems about certain named persons or certain families or groups continued through many pages or days of work.

(6) They especially need individualization. For them both group and individual diagnosis and appropriate remedial work is called for, likewise there is need for differentiating materials of instruction for them, providing for the slow and for the more mathematically agile as well as the middle group.

(7) Their instruction should call for frequent reviews to offset the ravages of quick forgetting.

(8) Even more than the brighter group they need to be taught independence and the habit of checking results.

(9) The problem of their motivation is unusually important.

Care must be taken to avoid appeals to fear or worry or over-emphasis on marks or grades. Rather emphasis must be placed upon achievement and experiencing the satisfactions of being able to do things.

(10) They must especially be shown the values of and be given insight into uses of what they learn. There should be much application, much socialization for those of Track One.

Conclusion

I am not pessimistic about what the next few years will bring. Ten years ago I was much discouraged. Today I see the signs of awakening among the people who come to our summer schools, through the groups that meet in schools and teachers associations throughout the country through the year, and from the bookmen who meet mathematics teachers. The movement in the general direction of the trends that I described is gathering momentum. True, the application of these ideas in the schools is developing more rapidly in some sections than in others, but it is a nationwide movement. I think a great day is coming—a day like that in which I was in high school, when 80 to 90% of all students will be studying some form of mathematics. The question does not seem to me to be "if" but "how soon" and "who will be the leaders."

FLYING LABORATORY TESTS

A "flying laboratory" equipped with "tomorrow's radio and radar devices" has cut test hours by more than fifty per cent here at the Bendix Radio division of Bendix Aviation Corporation, it was disclosed today by W. L. Webb, director of fesearch and engineering for the company, world's largest developer and manufacturer of aircraft radio, and a top producer of radar and other precision communications and navigation equipment.

The specially equipped plane, a Lockheed "Lodestar" takes the air almost daily to test performance and airworthiness of such devices as automatic radio compasses, instrument landing systems and direction finders, and other new developments, Webb stated.

The airborne research laboratory acts as a double check on performance data obtained from simulated "life tests" with elaborate laboratory equipment and enables the division to explore new developments and verify their operating characteristics with a minimum of delays.

Flight research operations of the division are in charge of Ruel Colvin. Colvin, formerly a Design Engineer for Radio Research, predecessor company of Bendix Radio, pioneered first flight tests of the radio compass, now standard on military aircraft and world airlines. He is assisted by George Bevins, flight research engineer, formerly an instructor for Eastern Air Lines and in charge of flight research activities for the Sperry Gyroscope Company.

RADAR—THE INVISIBLE EYES OF WAR—WILL SAFEGUARD CIVILIAN LIFE IN PEACE-TIME

Robert N. Farr
Science Service Staff Writer

If you have ever shouted in the direction of a cliff, then measured the time it takes the echo to return to determine how far you are from the cliff, you have used a means similar to radar to check distance. However, radar uses ultra-high frequency radio waves, while the echo is made up of sound waves.

Many important Allied victories would have been virtually impossible without radar. In anti-aircraft defense, radar is used to detect the approach of raiding planes at great distances through darkness and fog. Installed on fighter planes, radar enables pilots to spot enemy planes in bad weather, and to get range for attack. It helped lick the U-boat menace by spotting submarines when they came to the surface at night to recharge their batteries. In spite of fog, smoke, or night's blackness, radar can spot the enemy more than 100 miles away. The day-after-day bombardment of Germany's arsenals and supply lines that preceded the invasion would not have been possible without radar.

When the war is over, radar will not drop out of the picture. Today we have only reached the bare beginning of radar development. Many peace-time applications are already known, others only need time for research to bring them into practical form.

When the war is over radar may be used to cut down railroad accidents. Radar units mounted in the engine cab of a locomotive would enable the engineer to detect oncoming trains on the same track, or trouble ahead, so that he could slow the train down in time. He would use the invisible eye of radar to give him visibility in storms, fog or on moonless nights.

Ships equipped with radar can sail into a harbor during a heavy fog and come into dock without colliding with other ships. At sea, radar will detect other ships, icebergs, floating wrecks, and other hazards.

In the air, radar will give pilots of commercial airliners an accurate picture of their altitude at all times. It will also detect objects such as high tension wires, radio antennae, tall buildings, mountains and other planes even though they are not visible, so that the pilot can steer clear of them. It will permit a plane to land in a dense fog, without other assistance.

Even your automobile may have a radar unit that will make driving safe in fog, storms, or snow. With a radar beam shooting out in front of your car you would know of the position of obstructions, other cars and trucks even though you cannot see them.

Until recently, it was taboo even to mention the word "radar," which means radio detecting and ranging: *ra* (radio) *d* (detecting) *a* (and) *r* (ranging). The letters r-a-d-a-r spell the same forward and backward. This spelling of the word gives a clue to what it is, a radio echo.

Twenty-two years ago, Dr. A. H. Taylor and Leo C. Young of the Naval Research Laboratory discovered that certain radio waves bounced back from steel, like the echo from a cliff. This was the beginning of radar for America as we know it today. Other pioneers rapidly picked up the idea and intensive research is still in progress. These men were Maj. Gen. Roger B. Colton, U. S. Army, Dr. John H. Dellinger, of the National Bureau of Standards, and Robert M. Page, of Naval Research Laboratory. Although these men were long on faith in radar, they were short on funds to carry on research.

As World War II came nearer to being a reality, radio manufacturers gave their cooperation in perfecting military radar, and in getting it into mass production. Today, military and naval men agree that we might have lost the war 10 years before it began, if these pioneers had not persevered in radar research.

The Axis got its first taste of radar from the United States on the night of Nov. 14, 1942. Out in the Southwest Pacific, off Guadalcanal, it was storming, and one of our warships was hunting for Jap men o'war. Like a searchlight beam, the radar beam probed the enshrouding turbulent darkness, until a reflected signal was received, registering the presence of an enemy vessel more than eight miles away.

Our big battleship raised her guns, and sent powerful high explosives thundering into the storm towards the spot where they knew the Jap ship lay. The second salvo landed squarely on the enemy man o'war, 14,000 yards distant. This experience vividly demonstrated radar's effectiveness, and soon afterwards compact radar units were being installed in airplanes, as well as on land and aboard ships.

How Radar Works

The fundamental principle of radar is that ultra-high frequency (very short) radio waves can be focussed and are reflected when they strike an object. These short electromagnetic waves travel at a rate of 186,000 miles a second, the speed of light. They are reflected from the object in much the same way as sound waves in an echo. The shorter the radio wave, the clearer the reflection. Radar waves are focussed at a definite object. They do not scatter as do the waves from radio in ordinary broadcast.

The transmitter and receiver rotate in unison so that they scan the same area at the same time. In this way, if a target or object intercepts the transmitted waves, it causes them to be reflected back to the receiver which is ready to pick them up.

The "key tube" of the transmitter is known as the directional klystron tube, perfected by Westinghouse engineers from an invention of Russell H. and Sigurd F. Varian, two brothers at Leland Stanford University. This tube, and others like it, projects the waves like a searchlight beam.

The radar receiver is a complex machine that receives the reflected waves and analyzes them. This machine measures the time between transmitting

and receiving the wave. The complexity of this instrument can be appreciated when you realize that these waves travel 186 miles, the distance from New York City to Baltimore, Md., in a thousandth of a second. Once the elapsed time is known, the instrument automatically records the distance.

Since you must know the exact position of an object in order to hit it with a bomb or bullet, radar shows the target's position. To do this, it uses a tube like the cathode-ray tube employed in television. Training the electron beam of the cathode tube on its fluorescent screen, the position of the target shows up as a spot of light.

The invisible radio waves used in radar can pierce darkness, clouds, fog, snow, rain. They can only be stopped by a solid body, which reflects them back.

Radar on airplanes has made possible precision bombing of a supreme quality. Bombing through the clouds from altitudes as high as 25,000 feet, is possible with the Pathfinder bombing technique, which uses radar, and smoke signals to locate and mark the target for bombers.

One plane, leading a formation of bombers, carries the Pathfinder equipment and a technically trained crew to operate it. When the bomber is over the target, the target shows up on the fluorescent screen. At this point, the Pathfinder plane drops colored smoke bombs or flares, leaving a ring of colored smoke in the clouds. With the target thus indicated, the bombers that follow merely release their loads within the colored ring, knowing that they will land on the target.

The Pathfinder technique is also used to land paratroopers. British planes, as well as the B-17 Flying Fortress and C-47 Skytrain, are used for this specialized kind of bombing.

So far as may be told without endangering our own forces or helping the enemy—this is the story of radar. Probably no instrument as competent as radar has ever been developed in such a short space of time. It is inconceivable now that we should have entered this war without radar. The fact that nearly a quarter-century ago scientists realized the value radar might have is evidence that in the postwar world scientific projects must be supported as insurance for our future security.

POSTWAR JOBS IN BEEKEEPING AND PHYSICAL THERAPY

Students, teachers, parents, vocational counselors and others interested in postwar jobs will find helpful information on opportunities in *Beekeeping* and *Physical Therapy* in two new occupational abstracts just published by Occupational Index, Inc., New York University, New York 3, N. Y. at 25¢ each.

In concise, readable form each abstract covers the nature of the work, abilities and training required, methods of entrance and advancement, earnings, geographical distribution of workers, advantages and disadvantages, as well as postwar employment prospects. The best references for further reading are recommended.

GROUP WORK IN HIGH SCHOOL MATHEMATICS

An Adaptation of An Elementary School Technique for High School Use

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How often is it the experience of a class in algebra or geometry that a large proportion of the class personnel sits idly by while a few give attention to teacher or pupil explanation of some problem or proposition? It is perhaps a rare teacher who does not find himself in this position now and again. But what if this is the experience of a class for a goodly part of the class period, several times a week, and so through the year? And when one ponders the aggregate waste of pupil learning opportunity which must be the consequence of such experience in hundreds of classes annually throughout the nation, the picture becomes very sobering indeed. Just how extensively this actually is the experience of classes in mathematics throughout the nation would be difficult to discover.

Certainly the disappointingly weak mathematics foundation recently reported[1] of so many of our service men will compel all of us concerned with effective instruction in this field to very seriously review our teaching techniques. There are, undoubtedly, a number of factors responsible for such weakness as revealed in the mathematics foundation of our youth, but it is the purpose of this article to touch upon but one of these, the efficiency of class work organization, and to suggest one instruction technique for addition to the teacher's repertoire of effective methods.

There are more ways of killing a cat than scaring it to death. Likewise, as every experienced teacher knows, there are a number of useful methods for securing to individual members of a class a profitable learning experience, varying in effectiveness, of course, with the teacher's skill and personality. There is one method, however, which seems to the writer to be worthy of special commendation to the teacher of mathematics, namely, organization of the class for group work. For, if one may so infer from the relatively small number of articles on group in-

[1] Dorwart, H. L., "Mathematics—Queen and Handmaiden," *School and Society*, Oct. 14, 1944, pp. 241–243.

struction reported in the Education Index for the last dozen years, it would seem that this technique has attracted far less attention than it deserves, especially at the secondary school level, and in the teaching of mathematics in particular. Incidentally, the writer may add, experience with this technique has demonstrated its value in other high school work besides mathematics, notably in foreign language instruction.[2] High school teachers of the social studies may also find this plan of organization profitable in their classes.[3]

The group instruction technique, referred to in this article, is a 'leaf from the book' of the first grade teacher of reading, who seems to have made most use of this procedure in recent years. In its operation, the class is divided into small groups of three to five pupils,—each group under the immediate direction of a student leader. The teacher thereby multiplies his guidance to a degree by the number of student leaders used. And, what is even more important, the number of organized work centers is also multiplied. Teachers will need no reminder that most individuals are inclined to be less reserved in small groups than in large, hence find in the group arrangement a set-up in which they are more likely both to seek and to secure early assistance to meet their needs. Several such small groups simultaneously at work in a classroom would seem certain to realize a greater amount of accomplishment per class period per pupil than is ordinarily possible when, for a good part of the time, there is but one center of attention for the entire class. Experimentation in this field, with a control group, would be interesting.

Determination of the personnel of each small group may rest with the teacher, who will doubtless be governed by considerations of pupil ability, personality, the probability of an individual profiting by or contributing to the group in question, etc. Or, at times, pupils may be permitted to group themselves in accord with their own preferences. Most teachers would probably wish to appoint the pupil leader for each group, but leaders may be changed as frequently as seems in the interest of either the group or the leader, or to give other pupils a leadership opportunity. The teacher may frequently wish to give special attention to either the weaker or stronger group, or to both, during a period.

[2] Kodaikanal School (Kodaikanal, South India), where the author served as principal during the years 1932–44, has used group work in its foreign language classes, Latin and French, very satisfactorily for the past five years.

[3] Henderson, K. J., "Organizing Group Work in the Social Studies," *American Childhood*, Jan. 1938, pp. 23–24.

The functioning of this form of class organization for mathematics instruction may be illustrated by the program which a class in algebra might follow, when once the group arrangement has been effected and understood, and can be readily put in operation by the class,—

(a) Beginning the class period,—Each group, preferably seated about a table rather than at desks, and removed as far from other groups as the room equipment will permit, will first correct their home-work for the day. To facilitate this correction, the teacher may provide each group leader with a list of answers, or simply write these on the blackboard. Each leader may also be provided with a check list of specific difficulties or points for mastery which are encountered in this assignment. Group and leader may be expected to review and discuss these items along with any other difficulties or questions raised by members of the group. At this time, too, some attention may be given within each group to the selection of advanced work for the next meeting of the class.

(b) Teacher assistance with special difficulties,—The teacher may confine her explanations of difficulties and principles to such items only as prove beyond the abilities of group leaders and their groups to handle satisfactorily. Often the teacher's assistance will be required only by one or two of the groups.

(c) Testing pupil progress,—After clearance of the difficulties encountered in the day's assignment, largely through group discussion, many teachers may wish to test pupil mastery, appreciation, attitudes, etc., by a five minute quiz, usually written. Some may make this practice almost a daily routine, for close check on pupil progress. These quizzes may be corrected by the pupils immediately, and so serve as one basis for determining the advanced work assignment.

(d) Assignment of advanced work,—About the beginning of the second half of the class period, the teacher may call for suggestions from groups or individuals regarding the work to be undertaken for the next meeting of the class, and immediately proceed to make this assignment definite. This, of course, would be the time for the teacher to offer counsel in methods of problem solution, or such guidance in procedures, understandings, or appreciations,

as the nature of the new work may necessitate, or which may be asked for.

(e) Attack on the new assignment,—In group arrangement thereafter, for the rest of the class period, pupils will individually attack the new work assignment, being free to discuss within the group such difficulties as may be encountered. The teacher will usually take this opportunity to give any needed study guidance to the slower pupils, as well as to offer an occasional challenge to superior pupils.

Some difficulties inherent in group work,—

(1) Group organization of class work is difficult to effect in classrooms equipped with the traditional desk-row seating. Small tables, not too high, with loose chairs about them, would be ideal,—a table for a group, of course.

(2) Each group must maintain sufficient orderliness and quiet to minimize disturbance of other groups in the room.

(3) Care should be exercised in the selection of group leaders to appoint those pupils whose personality is adequate for the task. An exacting domineering leader would be a liability always,—with group dissension only a question of time.

(4) Some additional work for the teacher would be involved in the preparation of material for group leaders,—e.g.: answer lists, and check lists of difficulties for mastery, as already mentioned; helpful guide material, briefly presented, for group reference in the attack on new work; etc.

(5) Pupil leaders will require some teacher guidance in the matter of giving assistance to other pupils,—not to permit themselves to be 'leaned upon' unfortunately, either for the growth of the pupil assisted, or so as to consume a regrettable amount of their own time.

(6) A non-cooperative pupil, when encountered, would probably be placed best with the teacher, instead of in a group, until his attitude improves.

It may be noted, finally, that the group arrangement of the class offers an organization which may, with a small amount of teacher guidance, function automatically on occasions when the teacher is called away from the class, or delayed in arrival, thereby maintaining the class program with minimum loss even in the teacher's absence.

NOTES FROM A MATHEMATICS CLASSROOM

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98. Learning How to Organize Details. Improvement in reasoning is generally considered the chief by-product of the study of geometry or one of the motives for studying geometry. Another important by-product is the growth in ability to organize and present some related facts. A class which limits the work to the study of theorems (whose proofs are presented in the textbook) and to simple exercises (like: if the diagonals of a parallelogram are 8 in. and 10 in., how far from each vertex is the point of intersection of the diagonals?) will never develop this ability. Often we set up splendid objectives in a course and then omit the material that promotes those ends.

When we are practicing "How to organize," the pupil should not have his energies or attention directed on "How to discover a proof" and so I furnish the proof. A good exercise is:

If triangle ABC is equilateral, CD is an altitude, and if the inscribed circle intersects that altitude at E, then CE equals a radius of the circle.

There are many possible proofs but for the purposes I have in mind I call the attention of the class to the fact in an equilateral triangle altitudes are medians, that in any triangle the medians intersect in trisection points, and if OD is a radius then CO is twice a radius, and so CE equals a radius. I have thus presented the proof in a conversational manner; the homework consists in organizing and writing the proof in the traditional formal way. The exercise, for the purposes here mentioned, must be such that the teacher can easily outline the proof in a few main steps. If the teacher furnishes too many details the pupils will merely repeat those steps. Further, the teacher should, if possible, avoid giving the ideas in the order in which they will be written in the proof, and should omit some important but easily overlooked detail. In the above example many pupils will overlook the fact that, to inscribe a circle, the angle bisectors not the altitudes are wanted. Just any exercise will not do, but there are plenty. And often I must rehearse what I plan to say to make sure that I do not tell too much or too little. A few examples are:

Through a point on the bisector of an angle draw lines parallel

to the sides of the angle. Then all the sides of the four-sided figure that is formed are equal.

If a quadrilateral is circumscribed about a circle, the sum of two opposite sides equals the sum of the other two sides.

The word by-product suggests incidental learning. Learning to organize material is too valuable to be trusted to chance. It should be sought, and taught, and considered as essential as learning that the base angles of an isosceles triangle are equal. To date we have found no way to test a pupil at the end of the year to see if he has acquired some of this ability except by the traditional method of asking him to solve an original exercise. None of the standard tests that I have seen test this ability. A test which presents all the steps of a proof in a disarranged order and then asks the pupils to rearrange them logically does not test all the ingredients that constitute organizing ability. It does not test ability to present something starting with a blank piece of paper, neither does it test his willingness to turn the pages of a book and hunt for what he wants.

99. Trigonometry and the Organizing Ability. The content of high school classes in trigonometry varies greatly and no published course of study would be a reliable indication of what is actually taught. In my own classes I estimate that about twenty per cent of the pupil's time and energy is spent on computations involving logarithms. The second week we solve right triangles—using logarithms; and each week thereafter we have a few problems requiring computations. About the fourth week we derive the law of sines and then use it for several weeks as the basis for problems. Thus the chapters on oblique triangles get spread throughout the semester instead of being treated on consecutive days. If logarithms have not been taught in the previous semester, I strongly advise the teaching of that topic during the first week of the trigonometry course. To solve right triangles without logarithms and then with logarithms is a waste of time.

Logarithmic work furnishes good material for developing ability to organize material and to present it in a clear way. As with geometric material, there is nothing to organize if the problem involves only a few steps. Like the puzzle problems of elementary algebra we must often use problems which sacrifice reality for purposes of instruction. Consider for example the following exercise:

In triangle ABC, $\angle B=90°$, $\angle A=26° 33'$ and $BC=34.56$.

On AC is drawn the equilateral triangle ACD. Find the area of the circle whose diameter is the altitude DE of triangle ACD.

The beginner finds log $AC=1.8883$ and then $AC=77.32$. Next, using a theorem of geometry or a trigonometric ratio, he finds $DE=66.96$, which is the diameter of the circle, and he can then find the area as required.

The class discussion should then point out that it is unnecessary to know the actual value of either AC or DE; only their logarithms need be known. Also, knowing that $\sin 60=\frac{1}{2}\sqrt{3}$ is useless information since it is easier to find log sin 60 in the table than to find $\log \frac{1}{2}\sqrt{3}$. Further, intelligent planning would derive a formula for the area before any of the logarithmic work is done.

$$\text{Area}=\tfrac{1}{4}\pi\left(\frac{a \sin 60°}{\cos A}\right)^2.$$

Likewise, a formula for the general problem should be derived before solving a specific problem like: Find the area of a regular polygon of 10 sides if a side is 3.456.

When the law of sines has been introduced, and A, B, and a are known, then

$$b=\frac{a \sin B}{\sin A} \qquad c=\frac{a \sin C}{\sin A}.$$

When such problems are considered for the first time, the normal pupil will begin by adding $\log a$ and $\log \sin B$. The more alert planner will find $\log a - \log \sin A$ first. Of course such problems must be assigned before the chapter on oblique triangles is read, for the textbook will solve a problem in the intelligent way. It is unfortunate that all texts in trigonometry deprive the student of a chance to do his own planning.

Problems which involve finding an altitude of a triangle when two angles and a side are known are good material for this kind of practice. They can be solved by using two equations, by using tangents, and by using cotangents. Typical of such problems is that of finding the height of a cliff from observations made at two points whose distance apart is known. My usual advice for such problems is: After solving the problem in one way, do it again some other way to see if you can improve the method.

Work with trigonometric identities is disappearing from many of the new texts. However, I find them useful when the

instructions are: Prove this identity in three or four different ways, and enclose in a rectangle the most intelligent way of working this exercise. Naturally I must assign fewer exercises if each one is to be done in several ways. I am not sure that the philosophers or educators who want schools to imitate life are right. When we leave school it is the correct answer to a problem that is important; in school, the methods of seeking an answer are also important. And, for purposes of instruction, the methods are more important than the answers.

100. Solid Geometry and the Organizing Ability. There is usually more than one reason for including any particular topic in a course. Solid geometry should be more than a study of mensuration. However, because it gives practice in organizing material, keeps alive computational ability, and makes the work seem intensely practical, I favor including problems like the following in a solid geometry class:

Find the cost of digging a ditch 8 mi. long if the cross section is a trapezoid 15 ft. wide at the bottom, 25 ft. wide at the top and 9 ft. deep, the cost being 35¢ a cubic yard.

Rain falls to a depth of 3.5 in. over a rectangular concrete court 42.5 ft. long and 24.75 ft. wide. If the rain is drained into a circular tank 12.25 ft. in diameter, how many inches will the water rise in the well?

The pupil is expected to present not merely a correct answer, but a solution that is well written; that is, any other worker must be able to understand how the answer was found. For the second exercise above, the work would be:

$$\text{Volume of rain on court} = \frac{3.5}{12} \times 42.5 \times 24.75 \text{ cu. ft.}$$

$$\text{Volume of rain in tank} = \pi \left(\frac{12.25}{2}\right)^2 \frac{h}{12} \text{ cu. ft.}$$

$$\pi \left(\frac{12.25}{2}\right)^2 \frac{h}{12} = \frac{3.5}{12} \times 42.5 \times 24.75$$

$$h = \frac{3.5 \times 42.5 \times 24.75 \times 4}{\pi (12.25)^2}.$$

$\log 12.25 = 1.0881$	$\log 14 = 1.1461$
$\log 12.25^2 = 2.1762$	$\log 42.5 = 1.6284$
$\log \pi = 0.4971$	$\log 24.75 = 1.3936$

log denom. = 2.6733	log num. = 4.1681
	log denom. = 2.6733
Water rises 31.24 in.	log [31.24] = 1.4948

How much detail should be presented in such work is a matter of good judgment by the pupil.

DEMONSTRATION LESSON USING THE FILM, "THE CORN FARMER"

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The following lesson is based on the Erpi film, *The Corn Farmer*. In this presentation the film is divided into four parts. This intermittent showing makes it possible to observe more closely and to check observations more carefully than is possible when a film is shown without interruptions. This technique necessitates a thorough study of the film on the part of the teacher and careful planning as to places in the film most suitable for stops and discussions.

During the discussions pertinent ideas may be recorded on the blackboard. This blackboard work is helpful in developing vocabulary and in using facts observed in the film. It places significant data before the group and furnishes facts for geographic thinking. It may be the basis for summarizing the lesson.

THE LESSON[1]

Introduction:

I want to take you on a Film Trip into the corn farming country. On this twelve minute trip we will become acquainted with Nelson White, a corn farmer, his wife, his son, and his daughter. We will see the work that this family does from spring through fall.

1. Let us see how many different kinds of work we find these people doing.
2. Let us see if this work is done by hand or if it is done by machine.

[1] Lesson given at Geography Section of Central Association of Science and Mathematics Teachers, November 24, 1944, using Seventh Grade Class from Washington Junior High School, Chicago Heights, Illinois.

3. Let us see how the work fits the season.

Show film through line, "Ed also feeds to the hogs skimmed milk from the dairy cows on the White farm."

1. What work did you see done?[2]
   Put on the Blackboard:

	M—Planting Corn
	M—Fertilizing the Soil
Spring Work	M—Cultivating Corn
	H—Feeding Hogs
	Corn
	Skimmed milk

2. What work was done by hand? We will put "H" in front of that work.
3. What work was done by machine? We will put "M" in front of this work.
4. How was the corn planted? About how far apart were the rows? (3 ft. apart).
5. Why does the spacing make it easy to cultivate?
6. What do they do when they cultivate? (cut down weeds, loosen soil)
7. What time of year do they plant corn? (spring—when there is no danger of killing frost) (Put *Spring Work* on the board.)
8. What time of year do they cultivate corn? (spring and early summer, until corn is tall enough to shade the ground.)
9. You saw the son, Ed, feeding corn to the hogs. I didn't see corn in the fields. Where did it come from? (last year's corn)
10. What else did Ed feed the hogs? (skimmed milk)
11. What other animals must this farmer raise? (dairy cows)

Show film through line, "Neither do I."

1. What kinds of work have you just seen?
2. Which of these were done by hand? Which by machines? Add symbols "H" or "M."
3. We saw mother and daughter at work in the house. What were they doing? (canning)

[2] Copy the *kinds of work* on the blackboard as shown. Note that the letter preceding each kind of work is added after questions 2 or 3; the season is added after question 7.

Add to the Blackboard List:

M—Cutting Alfalfa
M—Loading Hay Wagon (rake)
H—Balancing the Load (foot)
M—Filling Hay Mow
H—Feeding Sows
Summer Work M—Cutting Corn for Ensilage
M—Shredding Corn
M—Filling the Silo
H—Spreading the Ensilage
H—Picking Tomatoes
H—Canning Tomatoes

4. When is the canning season? (summer) Add ***Summer Work*** to blackboard list. When will the canned tomatoes be used? (winter)
5. What time of year was the alfalfa cut? (summer: July, August)
6. What did the film tell us was the best weather for haymaking? (broiling sun—hot, dry)
7. Why is alfalfa a good crop to rotate with corn? (roots which remain in ground after cutting contain nitrogen, enrich the soil)
8. What kind of corn is put into the silo? (green, moist—ears haven't ripened)
9. How is this corn prepared for ensilage? (stalks, ears and leaves are chopped together)
10. How is the ensilage put into the silo? (blower)
    When do they fill the silo? (late summer)
11. To what use do they put the ensilage? (feed for diary cattle, and other livestock)
12. When will it be used? (winter)
13. What does this suggest about the winters in this region? (severe)

Before showing the next part of the film:

You heard the White family say they were going into town. Let's go with them.

1. Let us see what kind of country we drive through to reach town.
2. Let us see what kind of place the town is.
3. Let us find the reason why the family went to town.

Show film through line, "Ed can make more money by fattening these calves and selling them than by selling the corn."

1. What kind of country did we pass through? (broad fields on both sides of the road, flat land)
2. Why is this good country for using machines? (big farms, level land)
3. What did we see in town? (courthouse, high school, stores, etc.)
4. What was the main purpose of the White family's trip to town? (buy calves)
5. How many calves did Ed buy? What did he pay for them? ($10.00)
   Where did these calves come from? (Colorado)
   What will be done with the calves on Mr. White's farm? (fattened)
   Why is this farm a good place for fattening them? (plenty of corn)
6. How were the calves taken to the White farm? (trailer on car)

Before showing the last part of the film:

We are now back on the farm. Again let us find the work the farmer is doing, how he is doing it, and how this work fits into the season.

Show film to the end.

1. What kinds of work have we just seen?
   Add to the Blackboard List:

	H and M—Husking Corn
	M—Hauling Corn
	M—Filling Corncrib
Autumn Work	M—Shelling Corn
	M—Grinding Corn into Meal
	H—Feeding Corn to Beef Cattle
	H—Loading Hogs Into Truck

2. How was this work done? (same symbols)
3. At what time of year was this work done? (autumn) Add *Autumn* to blackboard list.
4. What is a corncrib? (place to store corn)
   How does corn in a crib differ from that in a silo? (ears of ripe corn put in crib)

5. What is done with the corn that is put into the crib? (fed to animals in the winter)
6. What is sometimes done when the farmer has more corn than the corncrib will hold? (shelled or ground into meal)
7. How did Mr. White use most of his corn? (feed for his animals)
8. How were the hogs taken from the White farm? (by truck)
9. Where were these hogs going? (stockyards and packing houses—perhaps Chicago)

Completed Blackboard Work:

	M—Planting Corn*
Spring Work	M—Fertilizing the Soil*
	M—Cultivating Corn*
	H—Feeding Hogs*
	M—Cutting Alfalfa X
	M—Loading Hay Wagon X
	H—Balancing the Load X
	M—Filling Hay Mow*
	H—Feeding Sows*
Summer Work	M—Cutting Corn for Ensilage*
	M—Shredding Corn*
	M—Filling the Silo*
	H—Spreading the Ensilage*
	H—Picking Tomatoes
	H—Canning Tomatoes
	H-M—Husking Corn*
	M—Hauling Corn*
	M—Filling Corncrib*
Autumn Work	M—Shelling Corn*
	M—Grinding Corn into Meal*
	H—Feeding Corn to Beef Cattle*
	H—Loading Hogs into Truck*

Legend: H = Work done by Hand
M = Work done by Machine

* = Work pertaining to corn (symbol added during summary)

X = Work pertaining to feeding animals (added during summary)

Underlined items suggesting winter work (added during summary)

Summary—Now you have seen the entire film.

1. Who were the workers on this farm? (family)
2. Why was it possible for so much work to be done by so few workers? (machines)
3. Let's check our list to see how most of this work was done. Count the M's and the H's. (14 "M's" and 9 "H's")
4. Why was this farm well suited to the use of machinery? (large fields and level or gently rolling land)
5. Looking at the different kinds of work, let us check those that deal with corn. Put stars after the work that deals with corn. Count the stars.
6. Mr. White's farm had ideal conditions for growing corn:
   a. Days and nights were hot.
   b. Rain came in thunder showers.
   c. Soil was rich.
   d. Growing season was long enough to ripen corn.
7. How was Mr. White's corn crop used? (feed for animals)
8. What other crop helped feed his animals? (hay for horses) (Put X after work associated with hay.)
9. The work we have listed was spread through three seasons. Point to the seasons listed on the board.
10. From what we have seen, what leads you to believe that Mr. White's work does not end in the autumn?
    Underline the work that suggests *winter work*.
11. It is sometimes said that "Corn goes to market on the hoof." What does that mean? In this film did we see corn going to market in this way?

ELECTRONIC GLASSWARE

Electronic glassware, such as electric toasters, waffle irons, and other cooking utensils that can be put directly on flame, will be made after the war by a new process of molding glass with high-frequency electric current. The glassware is now used in wartime radio, radar, and other electrical instruments.

SOME LIGHT TRIGONOMETRY

Alan Wayne
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All too often, there are good applications which the physics teacher throws over to the mathematics teacher, who at the same time is shunting the problem back to the physics department. So between them the opportunity is dropped.

If mathematics be truly the language of science, then we should give our science students that means of expression, and our mathematics students such real applications, as correlation between the two subjects readily affords.

In "geometrical optics"—the most common ground of physics and geometry—there are mathematical applications which require little more than the scientific concept of wave motion, and the mathematical notions of the sine, cosine, and tangent, and their relation to the right triangle.

The mathematics student will readily accept the fact that light which comes from a point source generally ripples out as waves in all directions in space; but for a limited portion of space, the motion and wave form may be approximated as shown by the block diagram in Figure 1-b. The two ways of regarding the wave movement are shown in Figures 1-a and 1-c, respectively. The horizontal projection describes the wave disturbance by a *system of rays*, or alternately by a *system of wave fronts* (Figure 1-a). The vertical projection (Figure 1-c) is the curve that indicates the relative positions of vibrating particles of the wave. These conventions or points of view are too often not made clear to the physics student. However, these "physical" concepts are just as much "mathematical" in nature.

The average student who knows what a "sine" is will appreciate the derivation of Snell's Law when the physical picture is made clear. When a light wave passes from one uniform transparent medium to another across their interface, generally some of the light is reflected, as AQ, and some is refracted, as AA', in Figure 2. Letting v_1 and v_2 represent the velocities of the light in the respective mediums, in "unit time" the wave front ABC advances a distance v_1 in the first medium while the refracted front $A'B'$ advances a distance v_2 in the second medium. The index of refraction, n_1, of the second medium relative to the first, for that particular wave length, at the particular temperature and pressure conditions, is defined as v_1/v_2. But since i, the

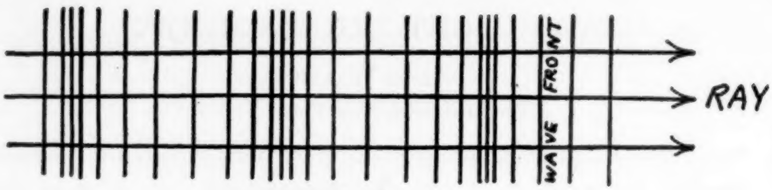

FIGURE 1 a

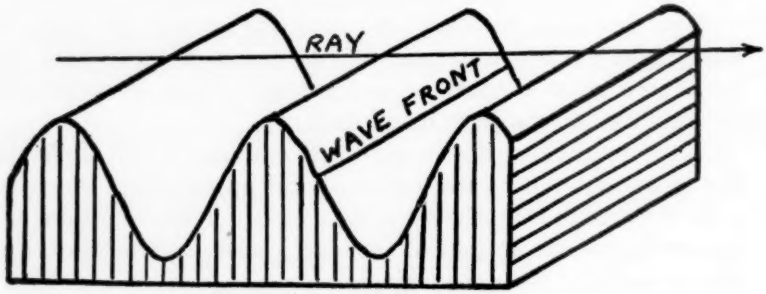

FIGURE 1 b

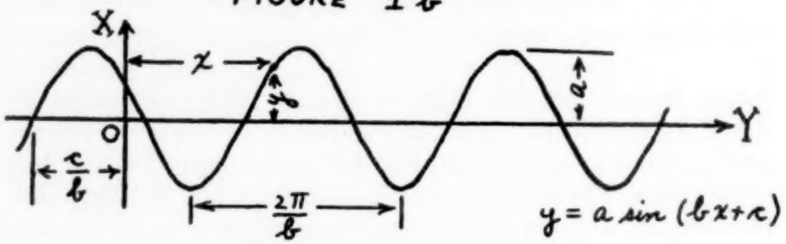

FIGURE 1 c

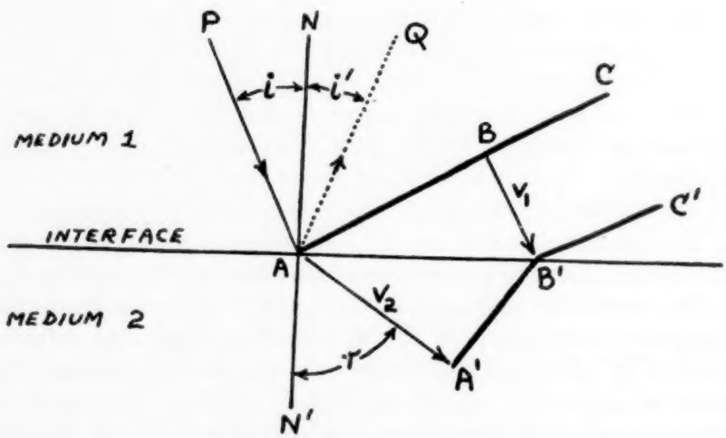

FIGURE 2

angle of incidence of ray PA, is also equal to angle BAB'; and r, the angle of refraction, is also equal to angle $AB'A'$, then $v_1 = AB' \sin i$, and $v_2 = AB' \sin r$. Thus $n_1 = \sin i/\sin r$, which is known as Snell's Law. If the velocity in the first medium is to be assumed greater than that in the second, the corresponding physical picture may be obtained from Figure 2 by omitting the dotted line AQ, while reversing the arrows on rays PA and AA', and interchanging numbers 1 and 2.

The student may now be asked to observe the changes in angle r as angle i increases (Figure 2). For a certain "critical angle" of incidence, it will be found that $r = 90°$, and this leads to the physical interpretation that for values equal to and greater than the critical angle, total reflection takes place. The physics student can confirm this result by experiment. Consideration of cases like this will emphasize the important fact that physical laws, frequently unlike their mathematical expression, are valid only within certain limits; consequently there must always be interplay of theory and experiment.

Light reflected, such as the ray AQ in Figure 2, from an uncorrupted interface, is more or less polarized in the plane of incidence. This means that the vibrations of particles producing the wave form (Figure 1-c) are in the plane determined by AQ and that line which is perpendicular to both AQ and AP. For a particular angle of incidence such plane-polarization is almost complete; and Brewster's Law states that the tangent of this "polarizing angle" equals the index of refraction of the reflecting medium (relative to the other medium). It is an exercise for the trigonometry student to show that in this case the reflected ray AQ and the refracted ray AA' are at right angles.

If the incident ray PA is already polarized in the plane of incidence, and PA (Figure 2) always makes the polarizing angle with the interface plane, then as the interface plane is rotated about PA from its original position (in which the interface plane is perpendicular to the "plane of the paper"), the intensity of the reflected wave AQ varies as the square of the cosine of the angle of rotation. This is known as Malus' Law.

More obvious is the relationship of the intensity of illumination of a beam of light and the angle of incidence of the light. If I_0 is the intensity of illumination of a beam of light on a surface normal to its rays, and I the intensity when the angle of incidence is i, then $I = I_0 \sin i$. In this instance the complement of i

will be recognized by teachers of "earth science" as the angle of solation.

A similar relationship is Lambert's Law of Diffusion, which states that when light is normally incident on a "perfectly diffusing surface," the intensity of the reflected light is proportional to the cosine of the angle of reflection.

The trigonometry student will have studied curves of the form $y=a \sin (bx+c)$, in which the constants have the significance shown in Figure 2-c. For a simple transverse light wave, such, as is usually considered, if y is the displacement of any particle at a distance x from the chosen origin, at a moment t by the clock, and a is the amplitude of the disturbance, and T is the full period of the wave (time between the passage of one wave crest and that of the next past a given point), and L is the wave length, then $y=a \sin 2\pi(t/T-x/L)$. Physical and mathematical significance may here be profitably contrasted by the mature student.

Other more advanced trigonometric-physical relationships are the diffraction-grating formula $L=(s/n)(\sin i+\sin d)$ in which L is the wave length of the incident light, s is the grating space, n the order of spectrum, and d the angle of diffraction; and Fresnel's formula for the ratio, R, of reflected light to incident light:

$$2R=\frac{\sin^2(i-r)}{\sin^2(i+r)}+\frac{\tan^2(i-r)}{\tan^2(i+r)}$$

These illustrations will have sufficed to show that the applications of trigonometry to light may range from the very simple to as complex as might be desired. Moreover, such correlations as those shown here may be extended to other fields of mathematics and science than those of trigonometry and optics.

Properly correlated, science and mathematics illustrate each other, and as a really valuable outcome of teaching, students will appreciate and understand the relationship between mathematical symbols and the realities for which they stand.

SCIENCE REVIEW

The *Science Review*, the science service journal of the Detroit group, for February–March contains some excellent articles on science, present and future, for all teachers of high school science. A card to Louis Panush, 3437 Oakman Blvd., Detroit 4, Mich. will insure you a copy.

THE VALUE OF MODERN LANGUAGES FOR CAREERS IN SCIENCE

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Science students are not all from Missouri, but as a rule they are the type that has to be shown—*why* certain subjects should be studied. Liberal arts students may take French so they can read the menus in ultra restaurants or converse with the maid they hope to have some day, while a science major wants to know what *practical* value he is going to gain from taking languages. At the stage when he should start studying at least French or German the embryo scientist generally has the vague idea that there are some pretty good scientists in Germany and maybe France, but what of it,—our country now leads the world in science and produces the best of everything, so why study foreign methods. Only after reaching the point at which all his time and interest are taken up by science, does the student realize how convenient it would be to be able to enhance his scientific knowledge by reading the foreign works.

Of course the Ph.D.'s are required to have a reading knowledge of two foreign languages, but these are a small minority of the technical forces, and furthermore, when they take their languages they usually want to get it over with quickly and work off the requirement with as little effort as is necessary to pass. Then, when plunged into the field of industrial research, the young scientist runs into one of the greatest arguments for multi-lingual reading ability—namely, patent study. American industry and research lives and progresses through its patents. They are the fruits from the millions of dollars spent by companies having research laboratories.

When a United States patent is issued it means that, as far as has been discovered, no like invention has been described in any language in any part of the world at any time since the beginning of scientific literature. Therefore, even if the worker has honestly and independently worked out a patent application, if some German or Patagonian (although none of the latter are known to be scientists) has described or patented the same thing in his country a hundred years ago, the work is in vain as far as patent protection is concerned. This means that at the very start of his work, before wasting a patent lawyer's time, to

say nothing of his own, or spending money on a patent application, the research worker must cover the world's literature as well as possible to see if his idea is new. This is not as difficult as it sounds, since, fortunately the literature is very well covered by encyclopedias and abstract publications, particularly in chemistry. A few hours, sometimes minutes, of searching will locate the references to be checked by complete reading or, if none are found, indicate a good probability that there are none.

This is where the languages come in. First of all, many of the best indexes are in German. The organic chemist's bible is "Beilstein" (*Handbuch der organischen Chemie*), a monumental work of 60 some volumes still in process of publication by the German Chemical Society. The main part gives brief mention of every piece of information published about any organic chemical from the beginning of chemistry to 1909. There is no substitute for this section, since there is no comparable work in any other language and no periodical abstract journal is complete. The first supplementary set, covering the literature of 1910–1919 and the second (about 20% complete) covering 1920–1929 are still the most convenient source for covering organic chemical literature. For the inorganic chemist the best reference set is another publication of the German Chemical Society—"Gmelin" (*Handbuch der anorganischen Chemie*), of the same scope as Beilstein, but covering all the literature up to a short time before publication of each installment, and now about 60% complete. However, there is a British counterpart of this set, published between 1922 and 1937, but it is not so well arranged or detailed as the German. Of course for recent years all branches of chemistry depend on *Chemical Abstracts*, but if nothing is found there, one can not be sure of completeness without cross-checking in *Chemisches Zentralblatt*, the German counterpart. Then if the indexes refer to a class of compounds or to a process which is close but not identical to the researcher's idea, someone must read the original work, in whatever language it may be, to make sure just what part of the idea is patentable if not the whole. If the reference is a recent one, there is a good chance that it will be in Russian, since a great deal of work has come out of the Soviet Union in the last fifteen years. If the reference is more than thirty years old, there is an even chance that it will be in German. No matter how old it is it must be checked.

It may be an injustice to the patent lawyers to tell the following, but it is a point which might fool many a scientist. In one

patent infringement case a chemical engineering consultant was testifying in behalf of the defendant by submitting as evidence that the plaintiff's patent was invalid, a book of 1870 publication date describing clearly the process in question. The opposing lawyer came right back with the argument that this book was too old and entirely out of date in the rapidly progressing field of chemistry. Of course the judge was not so ignorant of American patent law. This story also brings out the value of the literature to the company not having the patent but wanting to use the invention. If by covering the literature better than the patentee or examiner has done, an old reference to the invention can be discovered, the more diligent searcher may save a good deal of royalty money. And naturally, the references most apt to have been overlooked would be those in obscure foreign publications.

Perhaps we are straying from what the average scientist will need in the way of languages. There are always technical librarians and patent lawyers to do the detail work of verifying novelty or infringement. What the laboratory man wants is ideas. Very few of his ideas are the so-called "flash of genius." By far the greater part results from the desire for improvement or modification of something suggested by reading the work of others. Therefore, since science is international, the reading may involve any of the more than thirty different languages in which the scientific literature is published. This does not imply that the student should feel inadequately prepared if he can not read twenty or thirty languages. The law of diminishing returns takes effect after learning three or four. If the laboratory research worker can read the important journals in the languages most widely used for scientific publication, he will probably find all the ideas he has time to experiment with. Whenever he starts an experiment or laboratory project, the very first step is to learn all that has been accomplished in the field, then start from that point in the laboratory. This knowledge can be gained only by reading original articles. Summaries and abstracts usually feature the original author's conclusions or deductions, which may be partly or even wholly in error. The experimental facts and figures he has determined are quite apt to be reliable and they can be interpreted from the reader's point of view.

German is practically essential. Even if not another paper should come out of Germany in the next ten years, it would still be the most important foreign language because of the tremendous amount of literature published in the last one hundred

years or more. It is hard to realize the importance of the 19th century German literature until its use in practice is seen. A well known recent example of a gold mine found in the knowledge of the past is the new insecticide DDT, which was first prepared and described in 1874 by Zeidler, an otherwise unknown German, in the most widely circulated of German journals (*Berichte der deutschen chemischen Gesellschaft*). No doubt there are many other compounds waiting to be rediscovered and put to practical use. In the writer's relatively short experience in a research library, references as far back as 1830 have been of great importance in laboratory projects. So, if the Germans of a century ago were so far ahead of their time, who knows what might be possible from wider study of their current work.

For several years Russian has been in second place among foreign languages in science and after the war it is expected by many to be in first place. As has been stated by E. J. Crane,[1] editor of *Chemical Abstracts*, it has already replaced French in importance to chemists and will continue to be of increasing importance. Russian has long had the reputation of being one of the most difficult languages to learn. Perhaps so,—to speak, but to gain a reading knowledge for scientific literature is no more difficult than German, after the alphabet is learned. Although Russian has many difficulties peculiar to itself, it lacks many of the difficulties of German for an English speaking person. An article by J. W. Perry[2] should allay many of the fears of those who have long wished they had the time and courage to tackle the language of the "Russky zhurnals." When originally presented in the symposium on technical library techniques at the American Chemical Society Meeting in Cleveland in April 1944, it was one of the most popular papers on the program. Mr. Perry is planning at least one more paper in the series, in which he will present further short-cuts for those who have not had the opportunity to start from the beginning in school classes. Considering our recent relations with the Soviet Union, it would seem that there should be enough interest to warrant classes in Russian in the larger high schools. Certainly, high school students should be just as capable of learning it as German, and if the science students could take both then, it would give them a fine

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[1] Crane, E. J., "Growth of Chemical Literature; Contributions of Certain Nations and the Effects of War," *Chemical and Engineering News*, *22*, pp. 1478–1481, 1496, September 10, 1944.

[2] Perry, J. W., "Chemical Russian, Self-Taught. I. Suggestions for Study Methods," *Journal of Chemical Education*, *21*, pp. 393–398, August 1944.

start toward college work. For years scientists and educators have been saying Russian is the coming language in science, but no one seems to have done much about it. In view of the increasing quantity and improving quality of work in this language, research men can not much longer afford to pass up the abstract references because original copies would be too hard to obtain and translate.

At least one Romance language should be mastered for reading purposes. French is of diminishing importance but still the most important scientifically of the Romance group. The statistics of *Chemical Abstracts*[1] show an average since 1909 of about 10% of abstracts from French journals. Then too, some Swiss, Belgian and other authors publish in French, so there is too large a body of literature to be passed up in this language. Italian, Spanish and Portuguese follow in that order and of course each is easier to learn after studying the others. The latter two are apt to increase in importance as our South American neighbors continue to improve their universities and industrial laboratories.

One more language of some importance in the pre-war literature is Japanese. But fortunately, even the Japs recognize the difficulty of this language and much of their work is published in German or English, or at least has a summary in these languages. A very good picture of the contributions of various nations to the chemical literature is shown graphically by Crane in the article previously cited.

After the basic three—German, Russian and French, the laboratory scientist will probably have all he has time or urgent need for, but there is another field, open particularly to the girls, which offers great possibilities. Since this article or others like it will no doubt have little effect in turning out scientists with greater linguistic ability, there will always be a need for special librarians and literature researchers. These are the ones who can use any number of languages and should be experts in the basic three. The needs in this field and the training required are very well presented in an article by Egloff, Alexander and Van Arsdell,[3] another paper from the American Chemical Society symposia on technical library service. As mentioned before, the Ph.D.'s (who read French and German) are in the minority,

[3] Egloff, G., Alexander, M., and Van Arsdell, P., "Problems in Scientific Literature Research," *Journal of Chemical Education*, *20*, pp. 587–592, December 1943. This paper is from the first symposium in September, 1943, reported in the November and December 1943 issues. The second symposium was reported in the July and August 1944 issues.

and even they usually ask the library for a written translation when a reference is going to be used frequently or must be interpreted carefully. And of course the bachelors and masters, who usually have had a smattering of German but forgotten it through lack of use, require help continually. The Special Libraries Association, which runs an employment service as one function, had a total of 477 openings in 1944 for librarians and assistants, of which at least half were in technical fields. Practically all of the latter required a reading knowledge of French and German and the number requesting Russian is continually increasing. A few also requested Spanish and Portuguese. Since there are so few technical translation bureaus for the minor languages, most employers would probably say the more the better. Although "librarian" is usually thought of as a word of feminine gender, there is ample room in the technical library field for boys who have an interest in languages, since it requires long training and girls have an innate habit of marrying and leaving their careers just when well established.

Although over 40% of the chemical literature of recent years is in the English language,[1] and science is international, there is no immediate prospect of an international language. Thus, since we are still finding value in the scientific literature of over one hundred years ago, there will be no substitute in the forseeable future for translation of the work of the past if not of the then present.

NEW FLOATING REFRIGERATORS TO CARRY FOOD TO AMERICAN SOLDIERS IN SOUTH PACIFIC

There'll be ice cream, fresh meat, cheese, and eggs for American soldiers stationed in the Pacific war theater, brought to them in a new type of barge the Army has built for the purpose, the War Department reports. Three floating refrigerators, each costing $1,120,000 can store 64 carloads of frozen meats at 12 degrees above zero in the eight main holds. Two main deck compartments each have a capacity of about 500 measurement tons of fresh vegetables, cheese, eggs, and other perishable produce.

In addition, each barge has a special unit that turns out 10 gallons of ice cream every seven minutes and a plant that manufactures five tons of ice a day. The barge's elaborate cooling machinery is operated by 84 electric motors with capacities up to 150 horsepower. A complete change of arctic air is provided every four minutes to all chill and freeze compartments by 12 blowers.

The barges have flat-bottomed concrete hulls, are 265 feet long with a 48-foot beam and a 12- to 15-foot draft. They carry a crew of 10 men and 13 officers. The floating refrigerators will operate only in the southwestern and western Pacific. Small boats, operated by the Transportation Corps, will pick up the cargo and deliver it to troops, on an inter-island service.

WHAT IS GOING TO BECOME OF HIGH SCHOOL MATHEMATICS?*

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Not long ago a business man said to me, "What is going to become of high school mathematics after the war?" For reply, I gave him a stare of blank uncomprehension. I think I know the answer to the question, but it had simply never occurred to me that any rational human being would ask it. Receiving no immediate answer but that stare, he proceeded to elaborate his meaning. "For example," he said, "I have several times read the statement that when American, Canadian and British boys were placed side by side in training courses during this war, our boys were always poorest in mathematical ability. Of course, there were many criticisms of our mathematical achievements before the war, and the war seems to give point to these. After the war, are we going to drop mathematics as part of our program of general education? Are we going to alter the whole program of mathematics organization and teaching?"

I wish I could tell you teachers how well I stood up under this broadside, but, if you want the plain facts, I did not stand up at all. I could not think of the right answers. But I have had time to think it over and I wish to give you some of the answers I could only imperfectly think of under the pressure of that uncomfortable moment. I think that the statement that American boys do not compare favorably with Canadian and British in mathematics mastery is probably true, and furthermore I think this truth should have been anticipated. It is my guess that it will be true when we are forced to make such comparisons in any future like circumstances, and I think we should not be ashamed of the fact when we understand the reason for it.

The ideals of those who founded this country included equal educational opportunity for all, rich and poor, bright and dull alike. This ideal was written into state constitutions; it was preached from pulpits; it was shouted by orators; it was written into books; it was spoken in many a commencement declamation. About forty years ago this ideal began to "take" from one end of the country to the other. Academies, preparatory

* Presented at the Senior High School Group Meeting of the Central Association of Science and Mathematics Teachers, November 25, 1944.

schools, private schools became the people's high schools. The walls were burst; old buildings were torn down and new and bigger ones were erected, then additions to these were built. Tom, Dick and Harry, the sons of the farmers, small business-men and ditch diggers were going to high school, not to prepare for entrance into college, not to become professional men, not to be Gentlemen and Ladies, spelled with capital letters, but to learn things that would help them in the ordinary affairs of their lives. As a result, things began to happen to our schools. We had to organize courses differently and add new ones. We had to teach differently. We had to discipline differently. We had to revise standards of achievement. We devised intelligence testing, ability grouping, extracurricular activities, work books, visual aids, guidance. Some of us like all this, and some of us do not. What does it matter whether we like it or not? All this was tried out as a means of solving problems created or emphasized when Tom, Dick and Harry, Ruth, Jane and Mary crowded into our high schools. It is the greatest educational experiment of all time and it is succeeding. We shall not drop it, but we shall have to be prepared to pay a high price if we wish to see it through to the end. A part of this price is a little less perfect finished product. Those American boys who rated third best as mathematicians when compared with the products of other schools are but examples of this fact. But in place of a somewhat better mathematical polish, our American system gives them what it takes to be the best fliers and fighters in the world.

After the war, will there be high school mathematics? Will there be arithmetic, algebra, geometry, trigonometry? It is only when we forget our history or lose confidence in the lessons it has to teach us that such questions trouble us. As a measure of restoring our confidence, let us briefly review a bit of the development of elementary mathematics. Practical arithmetic was developed and perfected in response of the demands of trade and business. The business men of Phoenicia took the subject all over the world as they knew it and it found ready acceptance because it had much to contribute to the success and ease of doing business. The subject became and has remained through the centuries an integral part of the complete education of men. Has this war taught us that arithmetic has lost its value? Has it revealed that there is some subject better fitted to serve men in shop, factory and home? Has it shown us that we will be better off if we forget all means of knowing how much,

how many, how big when we buy and sell? No one would assert that the war has so changed our need for numbers and what men can do with them. The natural conclusion is that there will continue to be arithmetic.

And what about algebra? The Hindus, the Arabs, the Greeks, the mediaeval scholars slowly and laboriously developed and perfected the subject because they found it fascinating, and because it helped solve the problems of mensuration, demonstrative geometry, mechanics and astronomy. Other men made contributions to the perfection of the science because of that persistent intellectual nature of man which impels him to want to think God's thoughts after him, no matter to what end they may lead. With the coming of the technological developments of the later centuries, algebra became an indispensable tool of the engineer, the research scientist and the inventor. Has this war shown us that men are no longer interested in knowing for the sake of knowledge, or in acquiring the chief tools for building better machines, or in mastering techniques by which he can make scientific predictions and interpretations? It has not done this. Rather it has indicated the need for more power over algebra as a willing servant of men, and it is a safe conclusion that after this war there will be more algebra taught and learned than ever before.

Is the situation different with regard to geometry? The Egyptians developed geometry as earth measure. They used it to survey land; they used it to compute the capacity of grain bins; they used it to orient temples; they used it to build homes. The Greeks took over the subject and made it one of the foundations of logic. It became the first requirement for entrance to any university, and above the gate to the campus were carved the words, "Let not him who is ignorant of geometry enter here." The Greeks made it one of the chief means for training minds for clear, critical thinking. They studied the subject for its beauty, for its orderliness, for its power, and for its practical value. It spread all over the world and has held its place all these centuries because it still contributes all that it ever did, and somehow men sense this as a fact. Has this war taught us that we shall need less geometry? Are the shapes of things to come not to be geometrical? Will wheels no longer be circles? Will windows and floors no longer be rectangles? Will bearings no longer be spheres? Will bridges and buildings no longer depend upon triangles for their rigidity? Will we navigate ships

and planes without the principle of similarity? Will pistons and cylinders no longer be geometric solids? Will great circle sailing be out of date? Of course you know that no such revolutionary changes are to result from the experiences of this or any other war, or calamity, or golden age of prosperity, or new order, or new deal. We are going right on studying geometry for the same reasons we have always studied it, namely, because it has that to contribute which no other subject can offer.

But while high school mathematics will go right on after the war, there will come some changes in our method of determining who should take more of the subject and who should take less, how we shall teach it, what we shall emphasize, how much we shall attempt to cover in any given course, and it is these matters which are of most concern to us as teachers. The war has taught us certain lessons which to forget would be in the nature of a major educational calamity.

Lesson One. *Skills and information once acquired are soon lost unless there is a conscious effort to maintain them.* The musician knows this, the athlete knows it, the skilled mechanic knows it, but there are still a few teachers who have not found it out. We are too much inclined to believe that the pupil who has been taught to multiply and divide, to combine fractions, and to know when to apply the various processes has been given permanent mastery over these skills. The war has shown us that this is not true. The arithmetical incompetence of the young men entering the services and of the young women who took their places at bench and lathe was not due to the fact that they had never been instructed and drilled, but to the fact that there had been no systematic effort to maintain skills which they had once mastered. Businessmen have complained of this situation for a long time, but no one ever took their complaints as an indication of the existence of an emergency. But when there is a war to win or to be lost, we classify the loss of skills as an emergency and set out to do something about it. We set aside a few minutes a week in algebra classes to reteach and drill in the fundamental processes of arithmetic. In geometry classes, we set aside some time to reteach and practice on arithmetic and on the essentials of algebra. We test seniors to find out how much of once acquired skills has been lost and we follow this up with remedial instruction. We take and make opportunities in any mathematics course to use principles learned in other courses. Would it not be well for us to learn from this

experience that reteaching and practice on fundamentals are necessary in peace time as well as in war? Certainly we can find better ways to achieve this desirable end than we have used in response to war demands, but we should not assume that the problem itself will end with the war.

Lesson Two. *We need more emphasis on understanding and less on mechanical manipulation.* I know a teacher who mimeographed a list of rules from arithmetic and had pupils commit them to memory (a few at a time) and follow them in solution of problems. Of course, this is not as bad as the statement may seem, because he carefully developed each rule and formula when it was met for the first time, but I fear the pupils soon forgot the rational process by which the rules were developed and depended on memory and manipulation. Of course, there comes a time when all of the fundamental processes of mathematics become second nature to any of us and we perform them with a minimum of rationalization, as we should do. But the period of conscious rationalization should be much longer than is usually encouraged. The result of too early mechanization of processes is that pupils forget the rules and use the wrong one at the wrong time. There is the case of two sophomores in an engineering school who were in an argument about the result of adding a half and a third. One said the result was two-fifths because there was a plus sign between the fractions; the other said it was one sixth because, as he remembered the rule, one always multiplied the fractions. Neither was concerned with the fundamental principles involved but only with remembering the rule he had been taught.

Lesson Three. *In order to do a good job of teaching mathematics, we must become teachers of reading.* Most problems outside the class room and many in the text books are stated in words, either spoken or written. One of the major tasks in solving such problems is that of understanding what is said and implied as a means of translating the relationships into the language of mathematics. As you and I know, many otherwise good students fall down at this point. It does no good to criticize the teacher of reading for failure to prepare pupils to read mathematics. It is quite possible that she might do much better than at present. But the fact is that one never learns to read as a general achievement; one learns to read fiction, poetry, history, science or mathematics as more or less isolated achievements. No matter how well the pupil learns to read in the read-

ing class, he has to learn to read mathematics in the mathematics class. He has to learn to read between the lines; he has to learn to read with pencil and paper at hand, writing as he goes; he has to learn to read more than once for full comprehension; he has to learn to bring to bear mathematical principles previously employed. These are things that are mastered slowly, and they are things which you and I as teachers of mathematics will have to consider as part of our task.

Lesson Four. *We shall have to cultivate ability to judge as to the reasonableness of a result.* The person who solves a problem and gets an absurd result should be the first to recognize it as such. What some one else says about it, what the book answer is, whether he thinks he did it right, whether he feels that he made no mistake in his work should not weigh so heavily as his judgment as to whether the result is a reasonable one. A good way to cultivate this ability to judge as to the reasonableness of a result is to estimate the result before beginning to work the problem. Another practice that will insure correct results is the habit of checking every problem.

Of course all this takes time, and with courses of study to be followed, text books to be covered, college entrance requirements to be met, how is the teacher to find time for these other things? The obvious answer is the right answer: we shall have to reduce the number of topics taught, the number of methods and cases introduced, the number of problems solved. What will the colleges and universities think of this? They will approve without hesitation. During the past two years, it has been my privilege to talk with many college and university men in the mathematics and science departments of many colleges in Indiana, and have asked them what they think of this idea of reducing the amount of text book material covered. Almost without exception they are in favor of it ***provided we can thereby get the compensating advantages of better mastery.*** Please do not overlook the qualification. There is nothing to recommend doing fewer problems and topics merely as a means of making courses easier. But if it is a choice between doing fewer things better and doing more things less thoroughly, then there is no doubt we should pay the price of quantity for the reward of quality.

Discussions of more and better mathematics invariably raise the question as to whether we should raise mathematics requirements for graduation from high school. I think not. In Indiana and in other states and localities, the requirement is one year of

mathematics which may be algebra, or general mathematics, or vocational mathematics. I hope there will be no action, legislative or administrative, to increase this requirement for graduation. For college entrance or for other special needs local schools may well hold higher requirements for pupils preparing for special courses later. But consider what happens when the requirement is set up that every one must take three or four years of high school mathematics. Many pupils who have little ability or interest in the subject are in the classes with others who have high ability and expect to make definite use of the subject. The slower pupils suffer because they are forced to compete on unequal terms with the better pupils, and the latter suffer from having to take courses toned down to the level of slower pupils. Both lose. Of course, ability grouping helps greatly to solve these problems, but this does not furnish the answer to all the difficulties. It seems best to make mathematics elective after one year in high school, and trust wise guidance to find those who should and should not take the advanced courses.

Where mathematics is elective, you and I have an obligation that not many of us take seriously enough. We should all be salesmen of the subject, being careful not to sell it to the wrong persons. As teachers of mathematics, we are dealing with the most beautiful and powerful subject in the world. It is our privilege and duty to believe this. But our pupils cannot know this unless we reveal it to them. Take time out occasionally to reveal its beauty and its power. Use visual aids, puzzle problems, natural curves, biographical and historical material, linkages, mathematical games. Presumably this could be over done, but I have never known it to be so.

What is going to become of high school mathematics after the war? The answer is, it is going to increase in usefulness, it is going to be better taught, it is going to enter more than ever before into the most intimate affairs of men, it is going to contribute to our comfort, our culture, our usefulness. There is going to be more of it, not less. It is up to you and me to have faith in its possibilities, and, in this faith, to carry on.

A denial of opportunity to develop normally and naturally is responsible for an inferiority complex and that may be the forerunner of aggressivism resulting in delinquency.—JACOB PANKEN.

DENSITY PLUMMET THERMOMETERS

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The density plummets discussed before in this journal[1] are strikingly consistent in their reaction to change of temperature. So why not make a thermometer based upon the behavior of density plummets?

Following is a description of such a thermometer built to show whether a room is too cold, about right, or too warm. This

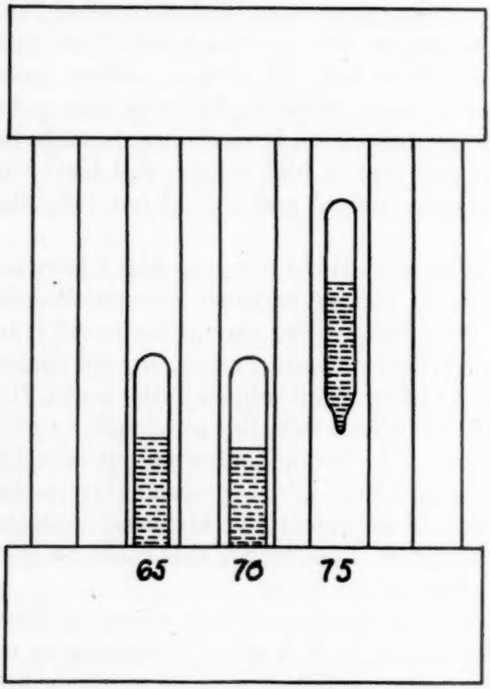

Fig. 1. The density plummet thermometer.

thermometer is designed to react at temperatures of 65°, 70° and 75° F.

Three plummets are made of very small test tubes, weighted to sink when the above temperatures are reached. (For directions see issue cited above.) The water inside is colored with blue ink. Each plummet is then put into a clean 15 millimeter test

[1] Lloyd, C. E., Density Plummets, School Science and Mathematics 44: 785–788, December 1944.

tube and the open end of the latter is heated in a Bunsen flame, using a blowpipe, until it closes to within 6 mm.

A short piece of 6 mm. tubing is welded to this hot end as a handle, and while the end is still soft a neck is pulled to a diameter of one or two millimeters. This gets rid of the thick glass which is likely to crack during the sealing process. The neck is cut short by using a carborundum crystal to crack it.

When the test tube is cool, fill it nearly to the shoulder with hot boiled water. Boiled water is essential because air in the water tends to collect on the plummet and change its buoyancy. Finally, seal the test tube with a blowpipe.

When the three units have been made, they may be put in an appropriate mounting. I put mine in two pieces of wood held together with two pieces of doweling. The upper wood is painted blue (symbolizing cold), and the lower is painted red (symbolizing hot). As a room warms up, the 65° plummet sinks first, then the 70° plummet, and if the room becomes too warm, the 75° plummet.

Pupils are enthusiastic about this thermometer. A dozen or so of fifty general science pupils are making their own. Almost any pupil can make plummets but only a few have the ability to make plummets which react at a desired temperature. It is a fine test of patience.

The final adjustment could be made by adding salt or alcohol to the water in which the plummet floats. I have not told my pupils of this method because I want to encourage skill in glass working.

Because the specific heat of water is high, the thermometer reacts slowly to changes in temperature. This lag can be shortened by using another liquid having a lower specic heat.

I have used both absolute methyl alcohol and carbon tetrachloride. Carbon tetrachloride has a considerably lower specific heat but it seems to have some effect on the glass which gives trouble when the test tube is being sealed. Plummets to be used in alcohol may be weighted with water, but those to be used in carbon tetrachloride must use something heavier. I use carbon tetrachloride colored with iodine. Perhaps shot or fine sand together with water would be easier to use.

Education is the only dictator that free men acknowledge, and the only security that free men desire.—Mirabeau B. Lamar.

GEOMETRIC GOLDBRICKS

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Coming off duty late one evening, I entered the Squadron Day Room with the intention of writing a letter home before retiring. In the corner a radio played softly and the few men present lounged in different positions, some reading, some listening to the music and others just dreaming. As I concentrated on the task before me, my thoughts were disturbed suddenly by a voice that said, "Tsk, tsk." Nothing more. Again it came. "Tsk, tsk." I glanced around and noticed a young GI hunched over some books at an adjacent table. He was apparently taking a correspondence course with the Armed Forces Institute. "Tsk, tsk, tsk," came the sound. No doubt about it, his was the voice.

"What's the matter, soldier?" I said. "Trouble?" He turned toward me.

"Plenty," he replied. "I'm trying to do some extra work, but I'm stuck."

In what appeared to be sheer desperation he handed me a book. A geometry text! Tenderly I placed it on the desk before me. It was almost two years ago that I had traded a ruler for a rifle. My brief reverie ended in a crash landing.

"Look," said my comrade in arms, "it proves that all triangles are isosceles and then asks me what's wrong. I'm stuck. To me it looks good."

"Well," I answered smugly, "let's go over it together. Maybe that way we can discover the error."

Simple, I silently said. This chestnut is as old as Euclid. Step by step we retraced the proof. Sure enough, all triangles are isosceles.

"Must have overlooked something," I muttered. "Let's go over it again."

Same result. All triangles are isosceles. To me it looked good, too! Two years ago I knew it was wrong, but that was long, long ago. Times were changing. Maybe back in the States all triangles are isosceles.

With despair in my voice I said, "Let me have your ruler and compasses." Laboriously I reconstructed the diagram from the book's statements. Hah! The given diagram was wrong. It all

returned to me slowly and vaguely. Triumphantly I pointed out the fallacy. All was right with the world—almost.

"Well, what do you know," my companion raged. "A goldbrick! They hand me a goldbrick and ask me what's wrong!" And mumbling under his breath, he gathered his material together and took off.

I leaned back in the chair. A goldbrick. The word had just the right sound. A goldbrick, Webster states, is anything purchased as valuable which proves to be almost or quite valueless. And I wondered how many geometric goldbricks I had sold each term!

The next evening I haunted the Base Library until I found a rather well worn copy of a text, and proceeded to review it with one purpose in mind—to discover how many geometric goldbricks were baked as the result of a construction made in achieving a proof. Attention was confined solely to the propositions, to prevent becoming lost in the profusion of exercises. Most instances were trivial. Those that follow are the more outstanding and instructive.

Goldbrick No. 1

The area of a parallelogram is equal to the product of its base by its altitude.

The proof of this proposition is dependent upon the construction of the altitudes from the upper to the lower base. The customary drawing is:

FIG. 1

The text proceeds to show that the area represented by K is equal to itself by identity, that triangle I is equal in area by triangle II and hence, that $I+II=III+II$ and so on to the Q. E. D.

Obviously, if the construction is valid, the proof follows. But, suppose that the given parallelogram is such that both altitudes fall outside the base. Thus,

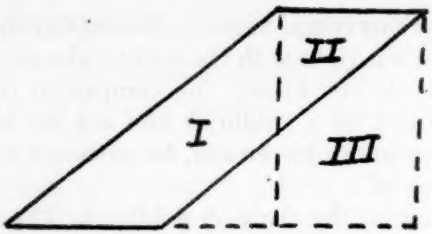

FIG. 2

Now the addition of *I* and *II* does not yield the given parallelogram, nor is *III*+*II* rectangular in form. Therefore, the proof as given does not hold.

Two methods for eliminating this objection suggest themselves. The first method is by making the statement (I offer no proof) that if the two altitudes to one side both fall outside the base, then by rotation of the figure to an adjacent side as base, one altitude will fall within the given parallelogram.

The second way consists in making the entire area of the figure identically equal and subtracting triangles *I* and *III* from it. The proof will hold regardless of how the altitudes fall.

By way of analogy, it should be pointed out that virtually every proof offered in Plane Trigonometry takes into account the position of the altitudes. The importance of such a procedure is evident here.

Goldbrick No. 2

In the same or equal circles, if two chords are unequal, they are unequally distant from the center and the greater chord is at a less distance.

Cor:—A diameter of a circle is greater than any other chord.

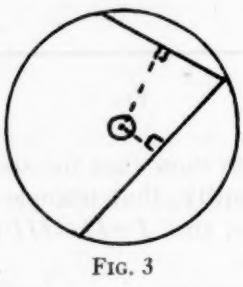

FIG. 3

The proof of the theorem is entirely dependent upon the possibility of constructing a perpendicular from the center of the circle to each chord. The construction is non existent for the

case of a diameter and the corollary is not a corollary in the true sense of the word. Actually, the statement should read:

In the same or equal circles, if two chords, not diameters, are unequal, etc.

The corollary should be proved first as an independent proposition. That is, in the figure,

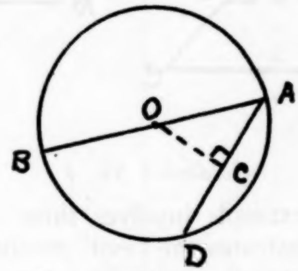

FIG. 4

the diameter may be shown to be greater than any other chord by making use of the statement that the perpendicular is the shortest distance from a point to a line (i.e.:—$AC < AO$).

The existence or non existence of a construction should be made a point for discussion. An example that comes to mind is the theorem, "The bisector of the exterior angle of a triangle divides the opposite side externally into segments which are proportional to the adjacent sides." Special mention is made for the case in which the bisector is parallel to the base. Furthermore, in the three dimensional analogue (the intersection of a plane and a sphere is a circle), the proof is divided into two parts as the plane passes through or does not pass through the center of the sphere. It deserves at least a mention here.

Goldbrick No. 3

Two triangles are similar if two sides are respectively proportional and the included angles equal.

The proof given, which is the usual one, is accomplished by superposition. Thus: (See next page).

Again, and this is typical of most similarity theorems, the exceptional case should be considered. If $B'C'$ should coincide with BC, then congrunece may be established. This in no way alters the relationship of similarity which has been reduced to the special case of a 1:1 ratio. In justification, it may be pointed out that this ratio is a common occurrence in every trade and technical field. Why overlook it in geometry?

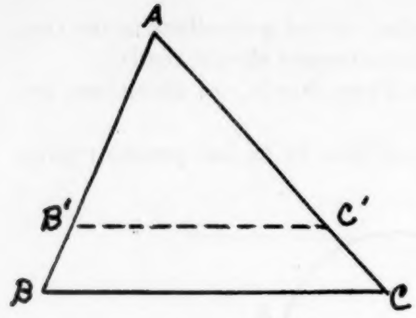

FIG. 5

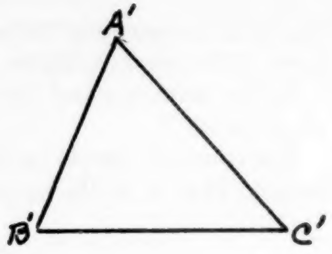

FIG. 6

Goldbrick No. 4

The concluding example involves three of the elementary propositions and illustrates the "evil" results of an indiscriminate use of the construction without a proper analysis. In the order given in the text, appended below are the three propositions, together with the outline of proof.

Part A. The base angles of an isosceles triangle are equal.

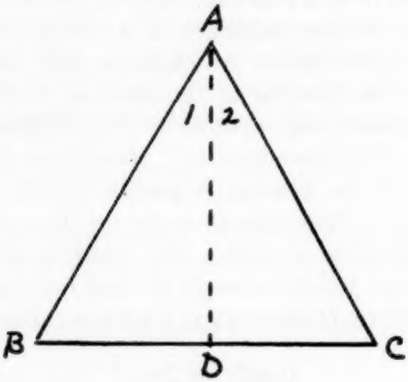

FIG. 7

1. $AB = AC$
2. $\measuredangle 1 = \measuredangle 2$
3. $AD = AD$
4. $\triangle ABD \cong ACD$
5. $\measuredangle B = \measuredangle C$

The demonstration is dependent upon the construction of the angle bisector AD. By means of this proposition, the text proceeds to show.

Part B. Two triangles are congruent if three sides of one are respectively equal to three sides of the other.

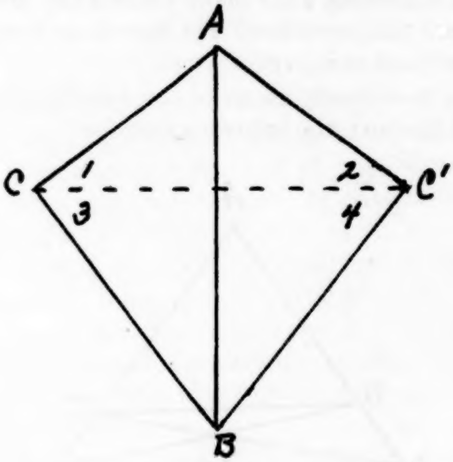

FIG. 8

1. $AC = AC'$
2. $BC = BC'$
3. $\sphericalangle 1 = \sphericalangle 2,\ \sphericalangle 3 = \sphericalangle 4$
4. $\sphericalangle C = \sphericalangle C'$
5. $\triangle ABC \cong \triangle ABC'$

Step 3, and therefore the theorem, is valid by virtue of Part A. The text continues on, to show

Part C. Required: To bisect a given angle.

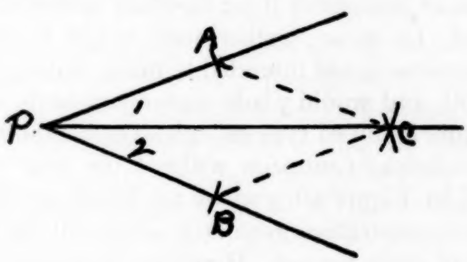

FIG. 9

1. $AC = BC$
2. $PA = PB$
3. $\triangle PAC \cong \triangle PBC$
4. $\sphericalangle 1 = \sphericalangle 2$ as required

The validity of the construction hinges on step 4 which has

been proved in Part B. But Part B depends upon Part A which in turn involves Part C. The result is a very fine example of circularity of reasoning. How many times a day are the students warned against this procedure? Yet, here is an instance in which it is offered without comment.

The author is evidently aware of this condition for he includes in the rear of the text, the following exercise:

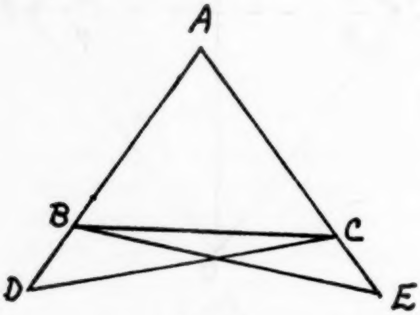

FIG. 10

Given: $AB = AC$; $DB = EC$
Prove: $\angle ABC = \angle ACB$

This proof provides the necessary sequence without the circularity. (Do I hear someone say, "Try and do it"?)

I do not contend that every construction should be made the subject of an exhaustive classroom study. Such a procedure would be almost impossible if for no other reason than the time limit imposed. To do so, furthermore, would be to ignore the fact that the course is not intended to make mathematicians out of every pupil, and would plade undue emphasis upon a comparatively more involved type of logic. The beginning student is faced with sufficient confusion without the insertion of problems typified by Figure 10, and we are all agreed that the presentation of demonstrative geometry is difficult enough without the addition of extra hazards. However, there can be no objection to the teacher making these investigations a part of the daily lesson plan, for both theorems and exercises, to be used if and when possible and desirable. This type of work may be used as an honor assignment for the brighter or more advanced students. In any event, somewhere along the line, they are worthy of at least a passing mention.

TEACHING VALUES OF THE PREPARED BIOLOGY DRAWING VERSUS THE ORIGINAL LABORATORY DRAWING

I. V. Tobler
High School, Loveland, Colorado

Ever since herbalists and zoologists began to draw, wood cut or engrave intriguing gross and minute biological structures, the succeeding generations of those of us who would instruct the uninitiated have generally sought to pattern our laboratory procedures on the practices of those ancients.

Whether drawing was really a psychological channel of learning was seemingly not questioned. There simply must be drawing done in laboratory. Not to do so would be considered heresy. However, after years of questionable benefits derived from this practice, there were those who began to look upon it with arched eyebrow and jaundiced eye.

These investigators, apparently becoming satiated with the practice of making the laboratory a combination art school and abbatoir, decided to see if it was really necessary for the biology student to spend endless hours on representative drawing. The result was that Fred L. Ayer and Olive E. Bryson conducted a bit of research. They found that "Those who made diagrammatic drawings knew more concerning the essential parts and their relations to one another, and were better informed about other items requiring discrimination than were those who did representative drawing."

These and other investigators did not bury the ancient laboratory drawing practice but had merely dug its grave. In other words, they showed that the time saving diagrammatic sketch was superior to the representative drawing, but did not indicate that this was so because of the time saving feature. It was supposed that something about a diagram made it a better learning tool than did a representative drawing."

This led to more research by A. M. Ballew and L. E. Taylor, who found in separately conducted tests that: "Pupils learn quite as much if they don't draw as if they do," which is really going all the way in discarding laboratory drawing. But then there were those of us who still believed in the value of well labelled drawings as an essential part of a notebook, so work was done to determine the relative effectiveness of laboratory drawing and the labelling of prepared drawings.

Morris L. Alpern, in a limited investigation covering a three weeks' study of the frog in an elementary biology course in The College of the City of New York, concluded that: (1) "The procedure of labelling prepared drawings and the procedure of making original drawings are equally effective"; and (2) "The two methods are equally effective at all levels of student ability."

The above deductions of Alpern are not in agreement with the findings of the writer of this article who made a much more comprehensive survey of students in a high school biology course dealing with the same problem. An attempt was made to conduct the investigations on more exact testing bases than appears to have been used with previous studies.

For this purpose, the Henmon-Nelson Tests of Mental Ability were administered the biology students, who were then transferred to one or the other of the two "guinea pig" divisions so as to place them on as nearly an equated basis as possible. This produced one group of 20 students having IQ's ranging from 84 to 124, and a median IQ of 104 and another group of equal size with IQ's ranging from 90 to 115 and a median of 105. The former group was assigned the task of doing its own laboratory drawings while the latter group labelled prepared laboratory drawings.

To determine accumulated biological fact prior to formal instruction, Form A of the Ruch-Cossmann Biology Tests was given in both divisions. After approximately nine months of similar instruction, with the exception of the differences stated above in laboratory procedure, Form B of the same test was given to ascertain the degree of achievement in acquiring factual knowledge by each of the two divisions.

During the same week in May that the Ruch-Cossmann Test-Form B—was given, the Iowa Every-pupil Test for Biology was also supplied the students. This test places major emphasis upon the functional values of the products of learning rather than on factual knowledge. Thus the two divisions were tested not only on their ability to accumulate biological facts but also on their ability to apply such knowledge in the solving of scientific problems.

What were the findings based on these tests? First, the two divisions started on a fairly equal basis as regards previously accumulated knowledge. Form A of the Ruch-Cossmann Tests showed that the division which was to do laboratory drawing

attained a median score of 16 points out of the possible 112, while the division that was to label prepared drawings had a median of 19 points.

After the approximate nine months of instruction, Form B of the same test showed the following results: The group that had done drawings dealing with 26 laboratory studies had raised its median from 16 to 46 for a median gain of 30 points; the group that had labelled prepared drawings covering the same studies had raised its median score from 19 to 61 for a median gain of 42 points.

On the Iowa Every-pupil Test the students that "drew" scored a median of 49 points out of the possible 99 while those that had "labelled" had a median of 54 points.

From the foregoing test results it can therefore be stated that: Where biology students are given the opportunity to label prepared drawings they do more effective work—not only from the standpoint of acquiring but also of applying knowledge—than if they are forced to do laboratory drawing. It is highly probable that gains made by the student in scientific knowledge are limited by the attention he must pay to the technique of making the drawing.

The achievement gain from labelling was greater among the students of lower ability, which would indicate that the less gifted pupil is greatly benefited by the use of the time saving prepared drawing. As for those of high IQ, it doesn't seem to make a great deal of difference either way. They learn in any case.

However, if a note of fatalism may be injected at this point, the time honored custom of preparing laboratory drawings that make the instructor cringe will probably be with us for a long time. And until a biological Moses comes along with a Heaven-sent tablet of Ten Commandments all of which say: "Thou Shall Not Draw," students will draw regardless of their IQ, the research findings of theses writers, and any law which might make the practice unconstitutional.

The author wishes to acknowledge the assistance of Dr. A. E. Holch of the Botany Department of Denver University in planning and carrying out the above investigation.

Bibliography

Alpern, Morris L., "A Comparative Study of the Effectiveness of Student-Made and Prepared Drawings in College Laboratory Work in Biology," *Science Education*, 20: pp. 24–30, Feb. 1936.

Ayer, Fred C., *The Psychology of Drawing*, pp. 107–68, Warwick & York, 1916.

Ballew, A. M., "A Comparative Study of the Effectiveness of Laboratory Exercises in High School Zoology With and Without Drawings," *School Review*, 36: pp. 284–95, April 1928.

Bryson, Olive E., "The Extent to Which Diagrammatic Drawing Contributes to the Understanding of Scientific Facts" (Master's Theses, University of Chicago).

Taylor, L. E., "The Ready-Made Drawing with Relation to Standard Achievement," *School and Society*, 32: pp. 371–74, Sept. 13, 1930.

PROBLEM DEPARTMENT

CONDUCTED BY G. H. JAMISON

State Teachers College, Kirksville, Mo.

This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.

All readers are invited to propose problems and to solve problems here proposed. Drawings to illustrate the problems should be well done in India ink. Problems and solutions will be credited to their authors. Each solution, or proposed problem, sent to the Editor should have the author's name introducing the problem or solution *as on the following pages.*

The editor of the department desires to serve its readers by making it interesting and helpful to them. Address suggestions and problems to G. H. Jamison, State Teachers College, Kirksville, Missouri.

SOLUTIONS AND PROBLEMS

Note. Persons sending in solutions and submitting problems for solutions should observe the following instructions.

1. Drawings in India ink should be on a separate page from the solution.

2. Give the solution to the problem which you propose if you have one and also the source and any known references to it.

3. In general when several solutions are correct, the ones submitted in the best form will be used.

LATE SOLUTIONS

1910, 11. *M. Kirk, Media, Pa.*

1912. *Felix John, Philadelphia, Pa.*

1904, 10, 12. *Pvt. Milton Scheffenbauer, Camp Wolters, Tex.*

1905, 7. *Morris I. Chernofsky, New York City.*

1903. *Proposed by Frank Brown, Linden, Michigan.*

Find the sum to infinity of the series

$$\frac{3}{1\cdot 2\cdot 4}+\frac{5}{3\cdot 4\cdot 6}+\frac{7}{5\cdot 6\cdot 8}+\cdots.$$

The solution presented here is a combination of solutions by Norman Anning, University of Michigan, Felix John, Philadelphia, Alan Wayne, Flushing, L. I., New York City, Milton Scheffenbauer, Camp Wolters,

Texas and Sam Morgan, Pasadena, Calif. Some of the solutions were rigorous, others quite simply stated.

By the process of separating the general term into its component fractions we have.

$$\frac{2n+1}{(2n-1)(2n)(2n+2)}=\frac{A}{2n-1}+\frac{B}{2n}+\frac{C}{2n+2}$$

where $A=(2/3)$, $B=(-1/2)$, $C=(-1/6)$, $B+C=(-2/3)=-A$, and $B=-3/4A$.

As a consequence,

$$\frac{3}{1\cdot 2\cdot 4}=\frac{A}{1}+\frac{B}{2}+\frac{C}{4}$$

$$\frac{5}{3\cdot 4\cdot 6}=\frac{A}{3}+\frac{B}{4}+\frac{C}{6}$$

$$\frac{7}{5\cdot 6\cdot 8}=\frac{A}{5}+\frac{B}{6}+\frac{C}{8}, \quad \text{and so on endlessly.}$$

The sum of the series to infinity can be written:

$$\frac{A}{1}+\frac{B}{2}+\frac{A}{3}+\frac{B+C}{4}+\frac{A}{5}+\frac{B+C}{6}+\cdots \quad \text{or}$$

$$A\left(1-\frac{3}{8}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\cdots \quad \text{or}\right.$$

$$\frac{2}{3}\left(1+\frac{1}{8}-\frac{4}{8}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\cdots \quad \text{or}\right.$$

$$\frac{1}{12}+\frac{2}{3}\left(1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\cdots \quad \text{or}\right.$$

$$\frac{1}{12}+\frac{2}{3}\log_e 2.$$

1915. *Proposed by Helen M. Scott, Baltimore, Md.*

In the loop given by the equation

$$\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^{2/3}=1$$

show that the subtangent of the tangent through the point of inflexion is equal to 2/3 of the abscissa of the point.

Solution by Brother Alfred, Napa, Calif.

Taken an implicit derivative:

$$\frac{2}{a}\left(\frac{x}{a}\right)+\frac{2}{3b}\left(\frac{y}{b}\right)^{-1/3}\frac{dy}{dx}=0$$

or

$$\frac{dy}{dx}=-\frac{3bx}{a^2}\left(\frac{y}{b}\right)^{1/3}.$$

Then:

$$\frac{d^2y}{dx^2}=-\frac{3b}{a^2}\left(\frac{y}{b}\right)^{1/3}-\frac{3bx}{a^2}\cdot\frac{1}{3b}\left(\frac{y}{b}\right)^{-2/3}\frac{dy}{dx}.$$

Setting this equal to zero for the point of inflexion, we find that at the point of inflexion:

$$\left(\frac{dy}{dx}\right)_i = -\frac{3y_i}{x_i}.$$

Hence the subtangent at the point is:

$$y_i\left(\frac{dx}{dy}\right)_i = -\frac{x_i}{3}.$$

A solution was also offered by M. Dreiling, Collegeville, Ind.; and the proposer.
The problem was not correctly stated.—Editor.

1916. *Proposed by Helen M. Scott, Baltimore, Md.*

Find the intersection, within the first quadrant, of the curves

$$\left(\frac{x}{a}\right)+\left(\frac{y}{b}\right)^{1/3}=1$$

$$\left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{b}\right)^{2/3}=1$$

when $a=8$, and $b=9/2$.

Solution by Brother Alfred, Napa, Calif.

From the first

$$\left(\frac{y}{b}\right)^{2/3}=\left(1-\frac{x}{a}\right)^2.$$

Substituting into the second:

$$\left(\frac{x}{a}\right)^{2/3}+1-\frac{2x}{a}+\left(\frac{x}{a}\right)^2=1$$

or

$$\left(\frac{x}{a}\right)^{4/3}-2\left(\frac{x}{a}\right)^{1/3}+1=0.$$

Let $z=(x/a)^{1/3}$. Then one solution is $z=1$, $y/b=0$. Factoring out $z-1$ leaves the equation:

$$z^3+z^2+z-1=0$$

By Horner's method an approximate root is found to be

$$z=.544$$

Hence

$$x/a=.161$$

$$\left(\frac{y}{b}\right)^{1/3}=1-.161=.839$$

or

$$y/b=.591.$$

Hence for the values of a and b indicated, the solution in the first quadrant is:

$$x=1.288 \qquad y=2.660$$

A solution was also offered by the proposer.

1917. *Proposed by Hugo Brandt, Chicago, Ill.*

In a circle of radius r inscribe a regular n-gon with side a, and a regular $2n$-gon with side b. For what n, is $a^2=r^2+b^2$?

Solution by Aaron Buchman, Buffalo, N. Y.

Given

$$a^2=r^2+b^2. \tag{1}$$

Let

$$y=180°/n. \tag{2}$$

Then it is easily shown by drawing the radii and apothems of the regular polygons that

$$a=2r\sin y \quad \text{and} \quad b=2r\sin(\tfrac{1}{2}y). \tag{3}$$

Replace (3) in (1) and

$$4r^2\sin^2 y=r^2+4r^2\sin^2(\tfrac{1}{2}y). \tag{4}$$

In (4), replace $\sin y$ and $\sin \frac{1}{2}y$ in terms of $\cos y$ and simplify. Thus,

$$4\cos^2 y-2\cos y-1=0 \tag{5}$$

and

$$\cos y=\tfrac{1}{4}(1+\sqrt{5}), \qquad \tfrac{1}{4}(1-\sqrt{5}). \tag{6}$$

From (6) it is at once evident (see division in extreme and mean ratio, construction of regular decagon) that

$$y=36°, \quad 108°. \tag{7}$$

From (2) and (7),

$y=108°$ results in no regular polygon,

$y=36°$ results in the regular polygons in which $n=5$ and $2n=10$.

Solutions were also offered by Brother Alfred, Napa, Calif.; H. M. Zerbe, Wilkes-Barre, Pa.; Felix John, Philadelphia, Pa.; Pvt. Milton Scheffenbauer, Camp Wolters, Tex.; and the proposer.

1918. *Proposed by Nellie Bishop, Waterloo, N. Y.*

If $a+b+c\equiv 0$, prove:

$$6(a^5+b^5+c^5)=5(a^3+b^3+c^3)(a^2+b^2+c^2).$$

Solution by M. Dreiling, Collegeville, Indiana

To prove: $6\Sigma a^5=5\Sigma a^3\Sigma a^2$, if $\Sigma a=0$.

(1) $$\Sigma a^2=(\Sigma a)^2+2\Sigma ab=-2\Sigma ab$$

since $\Sigma a=0$. Likewise,

(2) $$\Sigma a^3=(\Sigma a^2)(\Sigma a)-\Sigma ab^2=-\Sigma ab^2,$$

and

(3) $$\Sigma a^5=(\Sigma a^2)(\Sigma a^3)-\Sigma a^2b^3=2\Sigma ab\Sigma ab^2-\Sigma a^2b^3.$$

Substituting (1), (2), and (3) in the original equation, we obtain

(4) $$12\Sigma ab\Sigma ab^2-6\Sigma a^2b^3=10\Sigma ab^2\Sigma ab.$$

But $\Sigma a^2b^3=\Sigma a^2b^2\Sigma a-abc\Sigma ab=-abc\Sigma ab$. Hence (4) becomes, after division by Σab,

(5) $$2\Sigma ab^2+6abc=0.$$

Now

$$\Sigma ab^2 = (\Sigma ab)(\Sigma a) - 3abc = -3abc.$$

Therefore (5) becomes $-6abc+6abc=0$.

Solutions were also offered by Felix John, Philadelphia, Pa.; Brother Alfred, Napa, Calif.; Walter R. Warne, Marshall, Mo.; A. D. M. Lewis, S 1/c, Houston, Texas; M. Kirk, Media, Pa.

1919. *Proposed by Norman Anning, University of Michigan.*

If a and b are relatively prime positive integers such that $a+b=6$, 9, 12, 15, $\cdots$, then $x^a+x^b+1=0$ is factorable.

Solution by Howard D. Grossman, New York City

Let $w=\cos 120°+i \sin 120°$. Hence $w^3=1$.

$$f(x)=x^a+x^b+1=x^a+x^{3k-a}+1=x^a+x^{-a}+1.$$

If a is prime to 3, either $w^a=w$ and $w^{-a}=w^2$ or vice versa; similarly $(w^2)^a=w^2$ and $(w^2)^{-a}$ or vice versa. In either case

$$f(w)=f(w^2)=1+w+w^2=0.$$

Therefore

$$(x-w)(x-w^2)=x^2+x+1 \quad \text{is a factor of} \quad x^a+x^b+1.$$

The condition that a and b be prime to each other and hence to their sum $3k$ is unnecessary; it is necessary only that a, and hence also b, be prime to 3. e.g., x^4+x^2+1, x^8+x^1+1, and $x^{10}+x^2+1$ are also divisible by x^2+x+1 but x^6+x^3+1 is not.

1920. *Proposed by Julius S. Miller, New Orleans, La.*

Find the linear factors of $x^4+y^4+z^4-2x^2y^2-2x^2z^2-2y^2z^2$, and write the polynomial in determinant form.

Solution by Felix John, Philadelphia, Pa.

$x^4+y^4+z^4-2x^2y^2-2x^2z^2-2y^2z^2$ may be written thus:

$$\begin{aligned} &x^4+y^4+z^4+2x^2y^2-2x^2z^2-2y^2z^2-4x^2y^2 \\ &\quad=(x^2+y^2-z^2)^2-(2xy)^2 \\ &\quad=[(x^2+y^2-z^2)+2xy][(x^2+y^2-z^2)-2xy] \\ &\quad=(x+y+z)(x+y-z)(x-y+z)(x-y-z) \end{aligned}$$

The above result may be written as a determinant thus:

$$\begin{vmatrix} 0 & x & y & z \\ x & 0 & z & y \\ y & z & 0 & x \\ z & y & x & 0 \end{vmatrix}$$

A solution was also offered by Brother Alfred, Napa, Calif.

HIGH SCHOOL HONOR ROLL

The Editor will be very happy to make special mention of high school classes, clubs, or individual students who offer solutions to problems submitted in this department. Teachers are urged to report to the Editor such solutions.

PROBLEMS FOR SOLUTION

1933. *Proposed by Howard D. Grossman, New York City.*

Solve:

$$x^2-y=y^2-z=z^2-w=w^2-x=2.$$

1934. *Proposed by M. Kirk, West Chester, Pa.*

Find the sum to infinity:

$$1/3+1/3\cdot 3/6+1/3\cdot 3/6\cdot 5/9+\cdots.$$

1935. *Proposed by J. Frank Arena, Hardin, Ill.*

Solve for x, y, z:

$$\begin{aligned} x+y+z&=6\\ x^2+y^2+z^2&=14\\ xyz&=6 \end{aligned}$$

1936. *Proposed by Felix John, Philadelphia, Pa.*

Prove that $17^{2n+2}-288n-289$ is divisible by 82,944.

1937. *Proposed by Stella Williams, Ithaca, N. Y.*

Solve the system:

$$\begin{aligned} x^4-5xy^3+4y^4&=0,\\ x-y&=1. \end{aligned}$$

1938. *Proposed by Dorothy C. Hand, Clark's Summit, Pa.*

Find the exact lengths of the bisectors of the angles of the right triangle in which one leg is 1 and the hypotenuse is 3.

BOOKS AND PAMPHLETS RECEIVED

DICTIONARY OF ENGINEERING AND MACHINE SHOP TERMS, by A. H Sandy, *Silver Medalist, City and Guilds of London; Instructor and Lecturer, Mechanical Engineering Department, Borough Polytechnic, London.* Revised by I. E. Berck, Ph.D. Cloth. 153 pages. 13.5×21.5 cm. 1944. The Chemical Publishing Company, Inc., 26 Court Street, Brooklyn 2, N. Y. Price $2.75.

FUNDAMENTALS OF PHYSICS, by Henry Semat, Ph.D., *Associate Professor of Physics, The City College, College of the City of New York.* Cloth. Pages xii+593. 15×23 cm. 1945. Farrar and Rinehart, Inc., New York, N. Y. Price $4.00.

SOUL OF AMBER, THE BACKGROUND OF ELECTRICAL SCIENCE, by Alfred Still, *Electrical Engineer and author of Textbooks.* Cloth. Pages xiii+274. 13.5×20 cm. 1944. Farrar and Rinehart, Inc., New York, N. Y. Price $2.50.

TELESCOPES AND ACCESSORIES, by George Z. Dimitroff, Ph.D., and James G. Baker, Ph.D., *of Harvard College Observatory.* Cloth. Pages v+309. 14×21.5 cm. 1945. The Blakiston Company, 1012 Walnut Street, Philadelphia, Pa. Price $2.50.

THE COMET OF 1577, ITS PLACE IN THE HISTORY OF ASTRONOMY, by

C. Doris Hellman, Ph.D. Cloth. Pages 488. 15×22 cm. 1944. Columbia University Press, New York, N. Y. Price $6.00.

PLASTICS, SCIENTIFIC AND TECHNOLOGICAL, by H. Ronald Fleck, M.Sc., F.I.C. with Foreword and Revision of Chapter XIV by Carl F. Massopust, *Consulting Engineer*. Cloth. Pages x+325. 14×21 cm. 1945. Chemical Publishing Co., Inc., Brooklyn, N. Y. Price $6.50.

WARTIME APPLICATIONS OF MATHEMATICS, FOR USE IN JUNIOR AND SENIOR HIGH SCHOOLS, by John J. Kinsella, *Assistant Professor of Mathematics Education, The University School.* Pamphlet. Pages v+31. 15×23 cm. 1945. The Ohio State University, Columbus, Ohio. Price 50 cents. Liberal discounts for quantity orders.

RESEARCH TODAY, Vol. II. Winter, 1945. No. I. 15 pages. 21×28 cm. The Lilly Research Laboratories, Eli Lilly and Company, Indianapolis 6, Ind.

NINTH ANNUAL REPORT OF HUNTINGTON COLLEGE BOTANICAL GARDEN AND ARBORETUM, submitted by Fred A. Loew, *Director of Huntington College Botanical Garden and Arboretum and Head of the Department of Biology.* 47 pages. 14×21 cm. December 1944. Huntington, Ind.

EDUCATION OF TEACHERS FOR IMPROVING MAJORITY-MINORITY RELATIONSHIPS, Bulletin 1944, No. 2, by Ambrose Caliver, *Senior Specialist in the Education of Negroes.* Paper. Pages 64. 15×23 cm. 1944. Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C Price 15 cents.

BOOK REVIEWS

SOUL OF AMBER, THE BACKGROUND OF ELECTRICAL SCIENCE, by Alfred Still. Cloth. Pages xiii+274. 13×21 cm. 1944. Murray Hill Books, Inc., New York, N. Y. Price $2.50.

This is a science book for the layman on the progress of electricity from the beginning of electrical knowledge through the life of Faraday. That the author is thoroughly familiar with this phase of the subject is quite evident from the many references which only one who has made a rather exhaustive study of the subject would come across. He traces the investigations of the many men whose combined work gives us the understanding we now have of the fundamentals of electricity and the operation of electrical machines. In his discussion of the work of scientists he mentions many usually omitted. Most of us remember Jean Paul Marat as the "archbutcher of the French Revolution," and of John Wesley as the founder of Methodism, but Still pictures both of them as ardent students of electrical phenomena. His two chapters, "Amber and Lodestone" and "Lodestone and Amber" are largely the life and research of Michael Faraday. Yet near the close he points out that "The world knows more about Faraday than about Henry; but it cannot be denied that Henry was a great scientist and a deep thinker. He was handicapped in the matter of equipment, and had little opportunity to obtain experimental confirmation of his ideas. Schools and colleges, one hundred years ago, were institutions where heavily burdened teachers taught nearly all day long during the greater part of the year. Henry, at Albany, had little more than a month during the summer vacation in which to set up such apparatus as he was able to get together, and to conduct his experiments." Again in the

discussion of the Ruhmkorff coil he points out that "In Europe, the invention of the induction coil was attributed to Ruhmkorff. That is because in those early days, there was a tacit understanding that Americans were hardy pioneers, well able to defend themselves against the savage onslaughts of disgruntled Indians, but mentally inferior to the armchair philosophers of the Old World, whose afflictions were moral rather than physical. The honest historian, capable of smothering his prejudices and disregarding the turbulent claims of nationalism, must admit that a humble graduate of Harvard College, Charles Grafton Page, contributed far more than any European scientist to the development of the induction coil." In his final chapter, "The Legacy of Faraday," the author points out that "the scientist is hopeful. . . . The scientist is unselfish. . . . He is the perfect internationalist."

G. W. W.

Consider the Calendar, by Bhola D. Panth, Ed.D. Paper. 138 pages. 15×23 cm. 1944. Bureau of Publications, Teachers College, Columbia University, New York, N. Y. Price $1.25.

In this book the author briefly presents the historical growth of our calendar to its present form, points out the numerous changes that have taken place in the various countries at different times, and presents two possible solutions for producing a more nearly perfect calendar for the future. The historical overview shows the influence of occupational, political and religious influences on the methods of counting and recording time. The natural and arbitrary time units and their relations to historical developments are briefly discussed. The gradual change from a strictly lunar calendar to a solar calendar is briefly traced from the wondering tribes to those of fixed habitation in the valley of the Nile. Our present lunar-solar calendar thus had its origin, and has come down to us with the important changes made by the Caesars and later by Gregory, but still carrying the marks of the ambition of the Caesars with a short February and a long August, and our ninth and later months still bearing names that denote a year beginning with the Vernal equinox. Our present calendar should have numerous changes to make it suitable for modern business and science. The changes that have been proposed are presented and the decisions the leading countries have indicated to date are recorded. Had the countries of the world been able to agree the calendar change should have taken place January 1 of this year. January 1, 1950 is the next most suitable time. Will the conditions permit thought on the subject? This book gives briefly the underlying facts. It is brief but gives the essential material and should be widely circulated and read. It is very unfortunate that so important a book should have come out in paper binding.

G. W. W.

Metals and Alloys Dictionary, by M. Merlub-Sobel, Ph.D. Cloth. Pages v+238. 13.5×21.5 cm. 1944. The Chemical Publishing Company, Inc., 234 King Street, Brooklyn, N. Y. Price $4.50.

This is a "first" volume in this field. It is an attempt of the author to present a rather complete list of the metallurgical terms used by workers in this field. In many respects it is more than a dictionary. It not only defines the terms listed but in many cases gives a considerable amount of other information of great value to the worker. Physical constants, properties of chemical elements, alloys and raw materials used in metallurgy are defined and described. The discussion of such words as etching, malleability, metal gage, etc. consists of much more than mere definitions.

The same is true of the entire list of the chemical elements. Many expressions, such as Amco Furnace, are described and the firm name showing the origin (Amsler-Morten Co.) are given. Such symbols as A.P.I. for American Petroleum Institute and A.S.T.M. for American Society for Testing Materials are given. No doubt many omissions will be noted. The author admits this and asks that corrections be sent him for use in future revisions.

G. W. W.

HEALTH FOR YOU, by Katherine Bruderlin Crisp, *East High School, Denver Colorado.* A production of the Department of Instruction, Denver Public Schools, Denver, Colorado. Cloth. 14.5+20.5 cm. Pages xv+576. 194 photographs, tables 6, diagrams 32. J. B. Lippincott Co., New York. $1.80.

This excellent book written by a classroom teacher in this field in secondary education gives an approach to the subject which should show the high school student how desirable health is. She not only presents the knowledge of health but the presentation is such that the reader would want to practice good health habits. The profusion of pictures and diagrams illustrating the various items presented should make the textual matter more easily understood. The book is divided into 5 sections suitably subdivided. The first three sections are devoted to the discussion of personal health which to the adolescent is psychologically pertinent because good health improves personal appearance and youngsters are sensitive about personal appearance. Section 4 discusses community health and section 5 deals with safety. The plan of each chapter is as follows: 1—Questions which are often asked by students—the answers to which are found in the particular chapter. 2—The health habits, knowledges and attitudes that pupils should have before leaving high school. 3—A concluding statement. 4—A list of activities. 5—Tests designed to aid in estimating and understanding the chapter. Associated departments list and their relation to the topic of the chapter. Supplemental film and reference book list.

A. G. ZANDER

MAN AND HIS BIOLOGICAL WORLD, by Frank Covert Jean, Ezra Clarence Harrah and Fred Louis Herman, *Colorado State College of Education,* with Editorial Collaboration of Samuel Ralph Powers, *Teachers College, Columbia University.* Cloth. 16.5×23 cm. Pages viii + 629. Photographs 196, Drawings 85, Maps, diagrams and tables 36. Ginn & Co., New York. $1.80.

This book is designed as survey course in general biology for the college level. The context is composed of the following Units: 1. Life Became Associated with Protoplasm, which Requires a Constant Supply of Food. 2. Living Organisms Must Provide for the Perpetuation of Their Own Kind. 3. Living Things Must Adjust Themselves to Their Environments. 4. Science Has Made Great Discoveries Relative to the Nature and Control of Disease. 5. Synthesis and Decomposition Form a Cycle in Nature. 6. The Adaptations of Plants and Animals Have Been Seriously Disturbed by Man. 7. Living Organisms Have Evolved in Response to a Changing Environment. 8. Mendel Discovered Genetic Principles Which Make Possible the Improvement of All Living Species. 9. Man's Cultural Development Moved Slowly at the Outset but Has Been Greatly Accelerated by Science and Invention. These units are divided into 23 chapters, each chapter closes with a good reference list and about a dozen questions for study. Chapters 12 and 13 are very well written and treat the subject of

conservation. The chapter 9 on the endocrine system and chapter 11 on the metabolic cycle are also particularly well organized.

The author appears to have succeeded in interpreting and organizing biological material so that the undergraduate can obtain clear concrete ideas on biological phenomena within a reasonable space of time. The mechanical aspect of the book is excellent, good-sized type and non-glaze paper which is easy on the eyes.

A. G. Zander

A New Manual for the Biology Laboratory, by Bernal R. Weimer and Earl E. Core, *Professor of Biology at Bethany College, West Virginia, and Professor of Botany at West Virginia University, respectively.* Paper. Pages viii + 214. 150 drawings, 4 tables and 1 map. 23×28 cm. John Wiley and Sons, New York.

This is a manual for the high school level. The items covered by the manual are: Equipment, General Directions, The Microscope, Plant Structures (angiosperms), Food Manufacture (plant), Digestion in Plants and Animals. Food Transfer in Animals, Respiration, Excretion, Irritability, Vertebrate Skeleton, Muscles (frog), Reproduction and Development,Homology and Analogy, Heredity, The Animal Kingdom (Taxonomy), The Plant Kingdom (Taxonomy), Ecology.

A feature prominent in this manual is the outline for work in taxonomy; many biology manuals tend to devote very little space to this important phase of the subject.

The directions for work all appear consecutively over the first 49 pages, the other 164 pages are made up of good drawings ready for proper labelling by the student after his particular investigation, observations and recordings.

A. G. Zander

From Copernicus to Einstein, by Hans Reichenbach. Translated by Ralph B. Winn. Cloth. 123 pages. 15×23.5 cm. 1942. Alliance Book Corporation, 212 Fifth Avenue, New York, N. Y. Price $2.00.

The German original of this book appeared in 1927 as vol. 85 of a pocket-sized popular science series, "Wege zum Wissen," published by the Ullstein publishing house in Berlin. The 1942 English version is the literal translation of that little volume. This fact, it seems, should have been acknowledged by the author or translator, so as not to lead the reader to believe that he is getting "the latest word," that is, some recent slant, or recent developments in the theory of relativity. Aside from this—since the subject matter has not undergone marked changes since the twenties—the book doubtless fulfills its purpose as well as it did then: to give an intelligent general reader a fair idea of what the theory of relativity is about.

Written in non-mathematical, simple language it tells the by now familiar story of the rise of relativity as it grew out of centuries of dramatic interplay of observation, experiment, and theory. A first chapter deals with the shattering of the Ptolemaic cosmology through the work of Copernicus, Kepler, Galileo, and Newton. The second discusses the several theories regarding the nature of light, leading to the idea of the world ether as the medium of propagation of light waves. The third chapter, called The Special Theory of Relativity, tells the story of the unsuccessful experimental efforts to discover effects of motion through the ether, and explains Einstein's postulate of the constancy of velocity of light and its implications: the relativity of simultaneity, of duration, and of geometrical form. The fourth chapter discusses some earlier attempts, chiefly by

E. Mach, of treating non-uniform motion as relative. Chapter five, The General Theory of Relativity, explains Einstein's postulate of the equivalence of acceleration and gravitational fields, and discusses the physical consequences from it and the status of their experimental verification. The sixth, Space and Time, touches lightly upon the meaning and non-meaning of four-dimensional space-time, and of non-Euclidean curved space.

The author's ultimate interest and purpose seems to be to impress upon the reader the idea that the theory of relativity constitutes one of the great revolutions in human thought, comparable to the Copernican revolution in the 16th century, and that he, the reader, should show open-minded readiness to revise or discard his habitually acquired ideas about space and time in the face of the inescapable logic and conformity to facts of the theory.

How well the author succeeds in thus convincing his readers is hard to decide. The small compass of the book (111 pages of text in large print with very wide margins) with much time taken out to explain and illustrate elementary facts of physics and astronomy, and to philosophize about the ways of evolution of scientific thought in general, makes with necessity for only light surface treatment of the theory. Under such treatment, however, the theory cannot be made really intelligible. To mention only one point: would not a thoughtful "general reader" find it difficult to believe the strange things happening in "systems in motion" as contrasted to "systems at rest" in a theory whose very core is the relativity of motion? (This is not to question the theory, but only the presentation, or rather the possibility of doing justice to the theory in this kind of treatment.)

While handsomely printed the book is marred by many typographical errors which at one place render a whole passage from Newton unintelligible (p. 78).

There is one other point which, while of no consequence for this book, we should like to mention. In discussing Newton's work the author reiterates the traditional account of the reason for Newton's twenty years' delay in announcing his theory of gravitation: his scruples concerning a numerical disagreement between the force of gravity at the surface of the earth and at the moon's orbit, which disagreement disappeared when, twenty years later, Newton was able to use a newly determined value for the radius of the earth instead of the previously accepted, inaccurate one. This story, which had been questioned already by some 19th century British scholars, was conclusively disposed of in 1928 by Professor F. Cajori. In a searching historical study he showed (a) that the widest range of values for the radius of the earth was current in Newton's time, and readily accessible to him in well-known publications; and (b) that the true obstacle for the completion of his theory (begun in 1665) was the problem of calculating the gravitational attraction of the earth—an extended solid—on a point on or near its surface, which problem Newton solved only in 1685. (See F. Cajori, "Newton's twenty years' delay in announcing the law of gravitation," in *Sir Isaac Newton, 1727–1927, a bicentenary evaluation of his work*, the History of Science Society, Baltimore, 1928, pp. 127–188.) It would seem well, therefore if this Newton legend, appealing and dramatic though it is, would begin to disappear from serious literature.

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When you change address be sure so notify Business Manager Ray C. Soliday, P.O. Box 408, Oak Park, Ill.